DESIGN AND IMPLEMENTATION OF A SOFTWARE TOOL FOR DAY-AHEAD AND REAL TIME ELECTRICITY GRID OPTIMAL MANAGEMENT AT THE RESIDENTIAL LEVEL FROM A CUSTOMER'S PERSPECTIVE

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AMDG

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SUMMARY

This thesis focuses on the design and implementation of a software tool able to achieve electricity grid optimal management in a dynamic pricing environment, at the residential level, and from a customer's perspective.

The main drivers encouraging a development of energy management at the home level are analyzed, and a system architecture modeling power, thermodynamic and economic subsystems is proposed. The user behavior is also considered.

A mathematical formulation of the related energy management optimization problem is proposed based on the linear programming theory.

Several cases involving controllable and non-controllable domestic loads as well as renewable energy sources are presented and simulation scenarios illustrate the proposed optimization strategy in each case.

The performance of the controller and the changes in energy use are analyzed, and ideas for possible future work are discussed.

CHAPTER 1

INTRODUCTION

A *home grid* can be defined as a small-scale energy system operating at the home level.

Today, home grids are essentially made up of loads which are most of the time manually controlled: lights, TV, washing-machine, etc. Additionally, electricity tariffs for residential customers are in most cases constant, or piecewise constant.

The future penetration of distributed generation, energy storage, and controllable loads located downstream of the meter, along with the likely development of dynamic pricing policies, will make energy management a more complex optimization problem which may be very difficult for the common user to solve. Residential customers will need the assistance of management software to optimize their consumption and react to varying price and incentive signals in order to minimize their electricity bill.

This thesis proposes an optimization strategy to achieve electricity grid optimal management at the residential level, from a customer's perspective. This strategy involves the modeling of power, thermodynamic and economic systems, as well as user behavior modeling, and can operate in both day-ahead and real-time, acting as the brain of the home grid.

The benefits of the proposed optimization strategy are:

- to help the residential customer minimize his electricity costs by creating a tool which can achieve home grid optimal management.
- to help utilities reduce the peak demand which results in energy and cost savings by enabling the residential customers to respond to day-ahead price signals and real-time consumption reduction requests.
- to motivate the deployment of smart grid technologies at the residential level, including distributed generation and storage, intelligent appliances, and smart meters enabling dynamic-pricing, and to enable the optimal management of these technologies.
- to propose a model for residential customer behavior which could be included in future multi-agent based simulations.

The rest of the thesis is organized as follows. Chapter 2 gives an overview of the main drivers encouraging a development of home energy management in the coming years. Chapter 3 proposes a modeling for the home grid system. Chapter 4 and 5 discuss the mathematical formulation of the optimization problem related to home energy management in two particular cases, and chapter 6 gives the mathematical formulation in the general case. In chapter 7, we extend the general case to a scenario involving several utilities, and in chapter 8 we specifically discuss real-time response. Finally, chapter 9 provides the conclusions of the present work and describes possible future work.

CHAPTER 2

ORIGIN AND HISTORY OF THE PROBLEM

2.1 - All-powerful utility vs. unresponsive customer

Since the two first electrical networks opened simultaneously in New York, USA and Bellegarde, France, in the Fall of 1882, electricity has become universally used at the commercial, industrial and residential levels. Today, electricity is certainly the most important energy carrier for human activity. Electrical energy can be produced from a wide variety of primary sources, including fossil, nuclear and renewable, and can satisfy multiple energy needs thanks to a distribution network relatively dense, efficient and reliable.

The immediate availability of electricity to the customer is considered in developed countries as part of our lifestyle, along with water availability everywhere and at anytime. However, contrary to water for instance, electrical energy storage is still difficult to achieve. Consequently, total generation must equal total demand at all times.

To facilitate the fulfillment of the above constraint along with the other capability limitations of the network, and due to the economies of scale of the electricity industry, electrical power companies have historically owned the whole infrastructure from generating units to transmission and distribution infrastructure, leading to natural monopoly situations.

In the early 1990s, various deregulation processes started reshaping the electricity markets separating the generation and retail functions from the transmission and distribution functions. Competition was progressively introduced at the generation level, establishing wholesale electricity markets where generation companies can sell their production to retailers. The transmission and distribution functions remained natural monopoly functions.

The ultimate form of competitive electricity market requires an additional market at the retail level where consumers can buy electricity from a pool of retailers. Retail markets have been established in some places, while the retail function remains a monopoly in some others.

Consumers are in theory able to consume electrical energy whenever they wish, the local utility being responsible to make sure that generation equals demand plus losses at all times, regardless of the amount of energy the customers are demanding. This is achieved through complex load forecasting estimations combined with frequency regulation using different types of resources:

- Online reactive reserves with automated controls that can respond immediately to maintain voltages within the required range.
- Online power sources with automated control that can respond instantly to minute-to-minute load fluctuations (regulation) and to major demand spikes (spinning reserves)
- Standby power sources with manual controls that can respond to demand spikes in 10 to 30 minutes.

In practice, situations where the demand starts exceeding the generation capacity may lead to power outage.

In return, utilities have a strong market power, especially when a retail market is not established, and are able to name their prices for electricity based on a regulatory process.

2.2 - Electricity pricing policies

Our human life cycle is divided into periods of activity and inactivity and this directly affects our energy needs. Consequently, the electricity demand curve as a function of time is not flat, but presents cyclic variations: the demand is low during the night, higher during the day, and very high in the evening.

These variations have an influence on the electricity prices which include the generation costs, and the utility margins. Indeed, the cost of generating electricity significantly increases during peak-demand periods because generating units used during such periods are more expensive to operate. Thus, the marginal cost varies during the day.

Along with the human life cycle, other factors may affect the electricity demand and prices : weather, season, public holiday, etc.

Today, most of the customers are not exposed to price variations because most of them have standard flat rate contracts with their utility. These contracts reflect the average price of electricity throughout the day. In some cases, electricity prices vary depending on the season.

Such static pricing policy does not prevent utilities from making profits, but make difficult to manage the demand during peak-demand periods, when energy-supply systems are constrained. Indeed, customers adjust their demand for a given type of good to the price variations of this good. In the case of electricity, they have no reason to modify their demand of energy because the price variations of electricity do not affect them in the short term.

2.3 - Existing demand side management programs and their limitations

Following the 1973 oil crisis, governments of many countries have encouraged the development of energy demand management programs. Demand side management consists of a set of actions that influence the quantity and/or demand curve profiles of energy consumed by end-users. Such actions particularly target reduction of demand during peak periods.

Demand side management programs require implementing a communication layer on top of the electrical layer. This enables the utility to send signals in advance or in real time to the end-users in order to influence their energy consumption.

Most of the existing demand response programs have been oriented towards large industrial or commercial customers, and are essentially load management based. Two main types of programs are in use, time-based and incentive-based programs:

- In time-based programs such as Time-Of-Use (TOU), Real-Time Pricing (RTP) and Critical Peak Pricing (CPP), the electricity price changes for different periods according to the electricity supply cost.
- In incentive-based programs such as Direct Load Control (DLC), Interruptible/Curtailable (I/C) Service or Demand Bidding, customers may receive incentives if they are willing to provide load reductions.

Some utilities also run demand management programs oriented towards residential customers. *Electricité de France* for instance, one of the world's largest utility companies, runs two programs based on TOU and CPP in addition to its standard flat rate contract to encourage its customers to shifts loads to off-peak periods.

- The day/night TOU-based program [1] proposes to customers above 6kVA a differential pricing between the day and night periods, with a fixed rate during the day (6am to 10pm) above the standard tariff, and a fixed rate during the night (10pm to 6am) below the normal tariff.
- The *Tempo* program [2] combines the previous day/night pattern with a CPP-based pattern. The days of the year are divided into three groups: blue (300 days), white (43 days) and red (22 days). Each category corresponds to a given set of rates for the day and night periods, blue days corresponding to economical rates, white days to average rates and red days to surcharged rates. The color of the next day is released online each day at 5pm.

However, the response of residential customers to demand management programs is usually very limited. In some places, residential customers simply cannot respond because they are only offered standard flat rate contracts. Additionally, the communication layer is most of the time inadequate, or simply does not exist, which prevents the utility to broadcast price signals to its customers, and to record their response.

Finally, when a residential demand management program is implemented, the decision process on the customer side is rarely automated, except in some cases for the heating system, which limits the customer's ability to respond.

2.4 - Consequences of the emerging smart grid technologies on the utility-customer relation

In its *Smart Grid System Report* published in July 2009 [3], the U.S. Department of Energy underlines that, in the smart grid environment, consumers will become an integral part of the electric power system:

« They help balance supply and demand and ensure reliability by modifying the way they use and purchase electricity. These modifications come as a result of consumers having choices that motivate different purchasing patterns and behavior. These choices involve new technologies, new information about their electricity use, and new forms of electricity pricing and incentives. »

Among the new smart grid technologies, several will have a direct impact at the residential level:

- *Distributed generation* -solar, wind, biofuel- enabling the customer to produce power. This will lead to bi-directional energy flows between the home grid and the utility grid.
- *Distributed storage*, enabling the customer to choose when to sell the energy he has in stock.
- *EV/PHEV vehicles* plugged-in to the home grid with power transferred back and forth.
- *Intelligent appliances*, able to communicate and coordinate with a local controller at the house level.
- *Advanced meters*, enabling bi-directional communication with the smart grid.

These technologies will change the utility-customer relation by enabling the residential customer to eventually become a *prosumer*, not only consuming but also producing power while interacting with the grid in a dynamic pricing environment. Prosumers will play an active role in the balancing process between supply and demand along with the utility, and more generally in adjusting operations based on grid conditions.

2.5 - Necessity of a local controller at the residential level

It has to be noted that some of the smart grid technologies have been around for a while. Differential pricing was first introduced by Landis & Gyr, a Swiss electricity meter manufacturer, which designed the first double tariff meter in 1903. Similarly, research has been conducted on electric vehicle or distributed generation since the 18^{th} century : Gustave Trouvé demonstrated a working three-wheeled automobile in 1881, and Charles Fritts created the first working solar cell in 1883.

The challenge today, from the customer's point of view, is to be able to *optimize* the use of these existing technologies, as well as all the emerging technologies which are to be deployed in the coming years.

In a smart grid environment, energy management at the residential level involves making economic choices based not only on the variable cost of electricity and the ability to shift load, but also on the ability to store or sell energy. And these choices have to integrate the customer's preferences in terms of comfort.

In that context, the optimal control of the home grid cannot be achieved manually requiring the assistance of a local controller, possibly embedded in the advanced meter.

This controller should be able to:

- integrate the customer's preferences, and the parameters describing the home grid
- analyze price signals broadcasted by utilities, and external factors such as weather forecasts
- make decisions to optimize the consumer's energy use in order to minimize the energy-related costs
- report these decisions to the customer, and to the various elements plugged-in to the home grid
- be flexible to fit a wide range of home grid configurations, and easy to use for the customer

The objective of this thesis is to propose an optimization strategy to successfully implement such controller.

CHAPTER 3

HOME GRID MODELING–
THE FOUR FUNDAMENTAL CONSTRAINTS

3.1 - Introduction

The proposed controller achieves home grid optimal management in a smart grid environment from a customer's perspective. In this context, achieving optimal management means to minimize the customer's electricity bill while meeting the customer's needs and meeting the constraints of the system.

The originality of this thesis is to encompass all the smart grids elements mentioned above -distributed generation, storage and smart appliances- as well as the customer's comfort preferences while being able to respond to dynamic price signals in both day-ahead and real-time situations.

3.2 - Day-Ahead Optimization

At the day-ahead level, this thesis recognizes the importance of the ability to shift the time when electric energy is transferred from the utility grid to the home grid. This allows the customer to respond to day-ahead price signals by planning to buy energy when electricity cost is low.

The time shifting of this energy transfer can be achieved in two ways: first, by hooking up a storage device to the home grid in order to decouple the timing of generation and consumption of electric energy. Over the last decades, electricity storage has made progress along with the development of renewable energies such as solar or wind which generate energy intermittently. Additionally, EV/PHEV vehicles may also be used in the future as storage resources during their vehicle-to-grid operation periods. The average vehicle is idle for 96% of the time [4] and usually parked at night at the customer's home allowing energy to be exchanged back and forth with the home grid.

In the following, we will assume the presence of a storage device connected to the home grid. The optimized management of this resource will be one of the degrees of freedom the customer will have to optimize his energy consumption.

The second way to enable energy transfer time shifting is load rescheduling. While with a storage system, timing of transfer to the grid and consumption are decoupled, with load rescheduling the energy transferred is used immediately and the storage system is not needed anymore.

In the following, we will assume the presence of reschedulable loads at the residential level, such as washing machine, dryer, dishwasher, refrigerator-freezer defrost, water-heater, heating and A/C, swimming pool heater and pump, etc. The clever scheduling of these reschedulable loads will be the second way for the customer to optimize his electricity consumption.

In conclusion, the customer will be able to shift the time when electric energy is transferred from the utility grid to his home grid by using both his storage system and load rescheduling.

3.3 - Real-Time Response

In the future smart grid environment, utilities will be able to broadcast every hour, every 15 minutes, or even in real-time signals to their residential customers. These signals will be either real-time price signals, or requests for electricity consumption reductions.

The later type of requests can happen during peak hours when the electrical grid is operating almost at its maximum generation capacity. In return, customers receive an incentive from their utility which is proportional to their energy consumption reduction.
Customers have two degrees of freedom to respond to real-time requests from utilities: curtailment/interruption of loads and augmentation of local generation. At the residential level, the heating and air-conditioning are the two loads which can be interrupted instantly. Generation units based on solar and wind are not controllable, but the use of biofuel-burning micro-turbines can give another degree of freedom to the customer. The use of energy previously stored in a storage device connected to the home grid is another option.

The choice to respond or not to a request is based on costs, and consumer's preferences. For heating and air-conditioning, preferences are expressed in terms of temperature over a given period of time. For biofuel-burning micro-turbines, the fuel cost is compared to the incentive proposed by the utility.

3.4 - System Modeling

The electrical system considered in this thesis consists of a set of solar photovoltaic modules, a small wind turbine, a biofuel-burning micro-turbine, an energy storage system, and a set of controllable and non-controllable loads (Fig. 1). In this section we describe how the various elements of the grid are modeled and integrated in the controller.

3.4.1- Solar Photovoltaic Modules

The power $P_i^{(s)}$ delivered by the photovoltaic modules over an interval of time d_i can be expressed as a linear function of the solar irradiance I_i over that interval, the total surface of the modules $S^{(s)}$, and the overall efficiency of the photovoltaic system $\eta^{(s)}$:

$$P_i^{(s)} = \eta^{(s)} S^{(s)} I_i$$

We will assume in the following that weather forecasts allow a day-ahead approximate estimation of solar irradiance and are available to the residential customer.

3.4.2- Small Wind Turbine

The power $P_i^{(w)}$ delivered by the wind turbine over an interval of time d_i can be expressed, in first approximation, as a function of the wind speed V_i, the air density ρ, the equivalent surface of the wind turbine $S^{(w)}$, and the efficiency of the overall system $\eta^{(w)}$:

$$P_i^{(w)} = \eta^{(w)} . \frac{1}{2} \rho S^{(w)} V_i^3$$

In reality, as soon as the wind speed reaches a specific limit -corresponding to the Reynold number (dimensionless) reaching 2500- the flow becomes turbulent, and non-linearities appear.

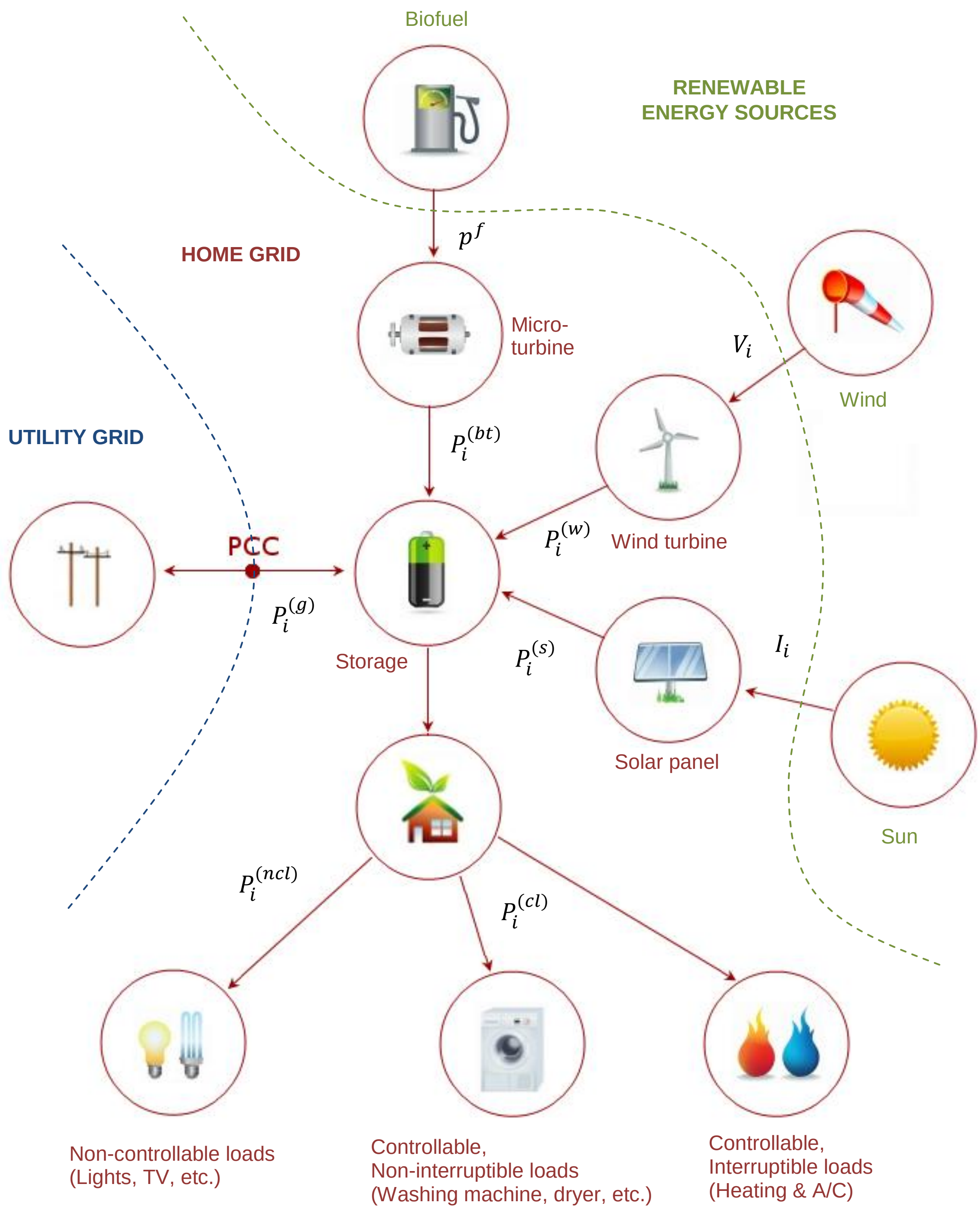


FIGURE 1: Home Grid – Electrical Grid Components

Although it is still difficult today to forecast with precision the wind speed because of its stochastic nature, we will assume in the following that weather forecasts allow a day-ahead approximate estimation of the wind speed and are available to the residential customer.

3.4.3- Biofuel-burning micro-turbine

The biofuel-burning micro-turbine is assumed to be fully controllable by the residential customer, delivering a power $P_i^{(bt)}$ over the time interval d_i.

In the following, we will assume that $P_i^{(bt)} = P_{nom}^{(bt)}$ when the turbine is turned on, with $P_{nom}^{(bt)}$ the nominal power of the micro turbine, and $P_i^{(bt)} = 0$ when the turbine is turned off. Also assume that p^f is the cost to operate in c/kWh.

3.4.4- Energy Storage System

Energy storage systems behave differently whether they are charging or discharging and their characteristics may also vary as a function of the number of charge/discharge cycles they have experienced.

In this thesis, we will consider a simplified model in which the storage system behaves like a water tank. Then, the behavior of the system can be described over a period of time $[t, t+1]$ by the initial amount of energy E^{init} stored in that system at t, the final amount of energy stored at $t+1$, and the maximum capacity E^{max} of the system. However, our methodology allows these to be extended to more detailed models in future studies.

3.4.5- Controllable and Non-Controllable Loads

From an optimization point of view, the residential loads can be separated into two groups, the controllable loads, and the non-controllable loads.

The non-controllable loads include all the domestic loads the customer wants to be able to turn on and off immediately: lights, microwave, TV, etc. It is desirable to inform the customer when he should reduce his use of these loads in order to minimize his electricity bill. But integrating these loads into a fully automated optimization process would be irrelevant as residential customers want to keep their ability to turn on certain loads such as lights, watch TV, take a shower or use their toaster whenever they need, including peak-pricing time periods.

One the other side, the controllable loads include the dishwasher, the defrost mode of a freezer, the AC, or the water heater tank. The customer is more flexible with the controllable loads as long as they fill their role. For instance, as soon as hot water is available at anytime, the customer does not mind if the water tank turns on at midnight or at noon in order to heat water.

Finally, the controllable loads can also be divided up into two subcategories, the interruptible loads, and the non-interruptible loads. The interruptible loads such as heating or air conditioning systems can be interrupted immediately whereas non-interruptible loads such as a dishwasher or a washing machine are not designed to allow unexpected interruptions when their cycle has started.

3.4.6- Grid Connection

The system considered is connected to the utility electrical grid, and power can be transferred back and forth to the grid. The electrical connection point of the home grid to the utility grid constitutes the point of common coupling (PCC).

3.4.7- Price signal, incentive signal

Day-ahead price signals are to be broadcast to the local optimal controller directly through the smart meter, or alternatively through a web portal. Utility companies charge the customer when power is transferred from the grid to his premises. We will assume that utilities have obligation to buy the power the customer is willing to send back to the grid at anytime. Buying and selling electricity prices may differ. Practically, a net metering system determines what the resulting cost is for the customer.

It is also assumed that incentives signals can be broadcast in real time to encourage customers to reduce their energy consumption.

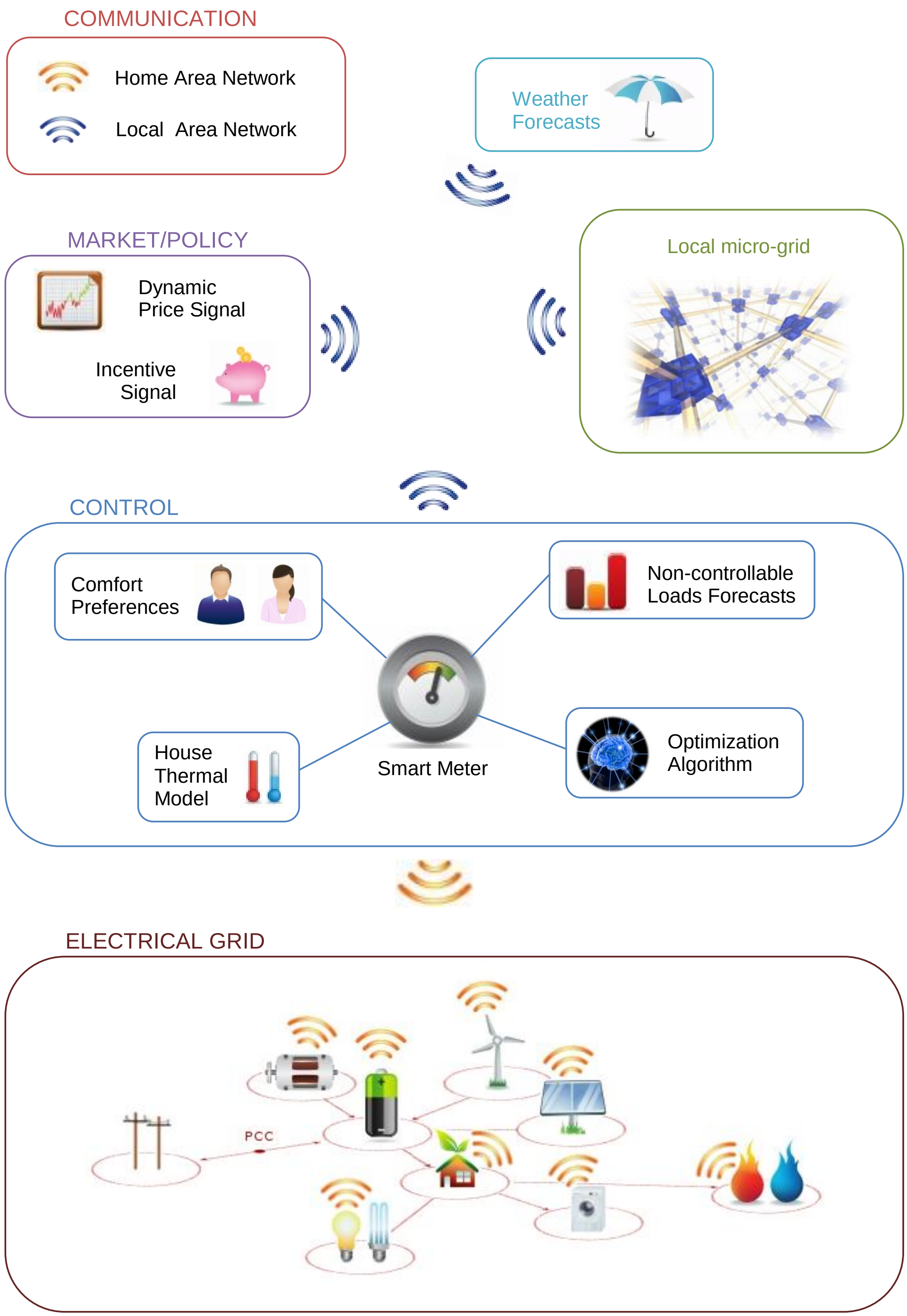


FIGURE 2: Home Grid - System Architecture

3.5- Mathematical Formulation

The energy E stored in the storage system is the state variable which will allow us to describe the state of the system throughout the next day.

E is a positive and continuous function defined on a period of time $\mathcal{T}_E \subseteq \mathbb{R}^+$. The values of E belongs to the interval $\left[E^{min},\ E^{max}\right] \subseteq \mathbb{R}^+$. The storage system capacity determines the maximum amount of energy E^{max} which can be stored in the system. The depth of discharge of the storage system determines the fraction of energy that can be withdrawn, hence the minimal amount of energy E^{min} which has to remain in the storage system at all times.

If $E^{min} \neq 0$, the behavior of the storage system can be modeled by a virtual storage system described by the energy function E_0, with $E_0^{min} = 0$ and $E_0^{max} = E^{max} - E^{min}$. Consequently, we can assume from now on without any simplification that E takes its values in $[0,\ E^{max}] \subseteq \mathbb{R}^+$.

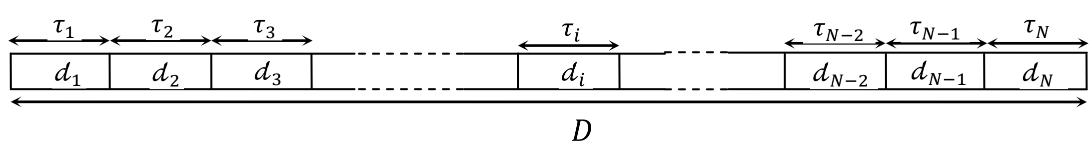
The variations of E must remain finite at all times as it is not physically possible for the storage system to exchange an infinitely great quantity of energy over an infinitely small time interval with the external environment. The physical characteristics of the storage system allow the definition of a maximal charging rate $R^c > 0$ and a maximal discharging rate $R^d > 0$.

The age and past history of the system, the number and frequency of charging and discharging cycles and the temperature may affect the characteristics and performance of the storage system. For simplicity, we will not consider these effects in the following, and assume that E^{max}, R^c, R^d remain constant over the temporal horizon during which the energy consumption is to be optimized.

In conclusion, four physical constraints apply to E :

$$
\begin{array}{lll}
\forall\, t \in \mathcal{T}_E, & 0 \leq E(t) & (1) \\
\forall\, t \in \mathcal{T}_E, & E(t) \leq E^{max} & (2) \\
\forall\, t_1, t_2 \in \mathcal{T}_E, & E(t_2) - E(t_1) \leq R^c & (3) \\
\forall\, t_1, t_2 \in \mathcal{T}_E, & -R^d \leq E(t_2) - E(t_1) & (4)
\end{array}
$$

Let D be the period of time over which we want to optimize the home grid electricity consumption. In day ahead optimization D usually corresponds to the next 24 hour period. Let us divide D into N adjacent time intervals d_i of length τ_i. Assume that the intervals have the same length such that $\forall\, i,\ \tau_i = \tau,$ and call $d = \{d_1, d_2, \ldots, d_i, \ldots, d_N\}$ a subdivision of D.



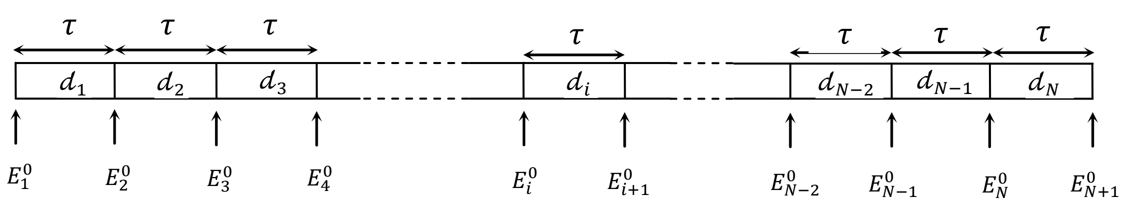
Assume for instance that D is 24 hour long. Then, if $N = 24$, D is divided into time intervals of $\tau = 60\, min$ each. $N = 48$ corresponds to intervals of $\tau = 30\, min$, $N = 72$ corresponds to intervals of $\tau = 20\, min$, $N = 96$ corresponds to intervals of $\tau = 15\, min$, etc.

For each interval d_i :

- E_i^0 is the amount of energy stored at the beginning of the interval
- $P_i^{(g)}$ is the power exchanged between the home grid and the utility grid with $P_i^{(g)} > 0$ when power is transferred from the utility grid to the home grid
- $P_i^{(bt)}$ is the power generated by the biofuel-burning turbine
- $P_i^{(s)}$ is the power generated by the photovoltaic modules
- $P_i^{(w)}$ is the power generated by the wind turbine
- $P_i^{(cl)}$ is the power consumed by the controllable loads
- $P_i^{(ncl)}$ is the power consumed by the non-controllable loads

The recursive relation between E_{i+1}^0 and E_i^0 is:

$$\begin{cases} E_1^0 = E^{init} & (5) \\ E_{i+1}^0 = E_i^0 + (P_i^{(g)} + P_i^{(bt)} + P_i^{(s)} + P_i^{(w)} - P_i^{(cl)} - P_i^{(ncl)}).\tau & (6) \end{cases}$$



The four physical constraints {(1), (2), (3), (4)} must be verified.

{(1), (2)} gives :

$\forall\, i \in [\![1, N]\!]$,
$-(P_i^{(g)} + P_i^{(bt)} - P_i^{(cl)}).\tau \leq E_i^0 + (P_i^{(s)} + P_i^{(w)} - P_i^{(ncl)}).\tau$

$\forall\, i \in [\![1, N]\!]$,
$\left(P_i^{(g)} + P_i^{(bt)} - P_i^{(cl)}\right).\tau \leq E^{max} - E_i^0 - (P_i^{(s)} + P_i^{(w)} - P_i^{(ncl)}).\tau$

With {(5), (6)}

$$\forall\, n \in [\![1, N]\!],$$
$$-\sum_{i=1}^{n}\left(P_i^{(g)} + P_i^{(bt)} - P_i^{(cl)}\right).\tau \leq E^{init} + \sum_{i=1}^{n}\left(P_i^{(s)} + P_i^{(w)}\right).\tau - \sum_{i=1}^{n} P_i^{(ncl)}.\tau \qquad (7)$$

$$\forall\, n \in [\![1, N]\!],$$
$$\sum_{i=1}^{n}\left(P_i^{(g)} + P_i^{(bt)} - P_i^{(cl)}\right).\tau \leq E^{max} - E^{init} - \sum_{i=1}^{n}\left(P_i^{(s)} + P_i^{(w)}\right).\tau + \sum_{i=1}^{n} P_i^{(ncl)}.\tau \qquad (8)$$

(α)

{(3), (4)} gives :

$$\forall\, i \in [\![1, N]\!],$$
$$\left(P_i^{(g)} + P_i^{(bt)} - P_i^{(cl)}\right).\tau \leq R^c - \left(P_i^{(s)} + P_i^{(w)}\right).\tau + P_i^{(ncl)}.\tau \qquad (9)$$

$$\forall\, i \in [\![1, N]\!],$$
$$-\left(P_i^{(g)} + P_i^{(bt)} - P_i^{(cl)}\right).\tau \leq R^d + \left(P_i^{(s)} + P_i^{(w)}\right).\tau - P_i^{(ncl)}.\tau \qquad (10)$$

(β)

Our objective now is to find the optimal controllable loads schedule -and biofuel burning turbine schedule- for the period of time D so that the electricity cost is minimized.

CHAPTER 4

THE NON-INTERRUPTIBLE LOADS CASE

4.1 - Introduction

We have divided the controllable loads into two groups: the non-interruptible loads (washing machine, dryer, dish washer, etc), and the interruptible loads (heater, A/C). Let us focus first on the non-interruptible loads in the absence of interruptible loads.

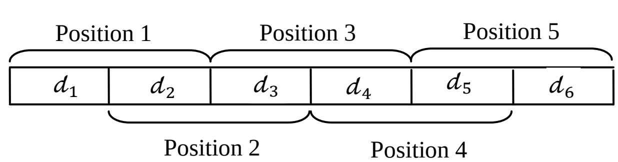
4.2 - Mathematical formulation as a linear programming problem

4.2.1- <u>Equality and inequality conditions</u>

Assume that the cycle of the non-interruptible load L is a multiple of τ, say $k_L\tau$. We must have $k_L \leq N$.

Then there are $S_L = N - k_L + 1$ positions possible to schedule the load L over D. Each position j corresponds to a binary number $\delta_j^L \in [\![0,1]\!]$. If the load L is scheduled in the position j, $\delta_j^L = 1$, otherwise $\delta_j^L = 0$.

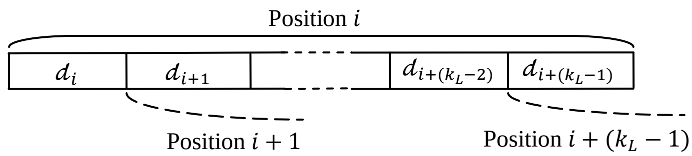
<u>Example</u> : for $N = 6$ and $k_L = 2$:



Let us assume that the load L is to be scheduled γ^L times over the next day. Then we must have :

$$\sum_{j=1}^{S_L} \delta_j^L = \gamma^L \tag{11a}$$

Additionally, we need to make sure that two cycles do not overlap.



We must have :

$$
\left.
\begin{array}{l}
\forall\ i \in [\![1, S_L-(k_L-1)]\!], \\
\qquad \begin{array}{c} k_L-1 \\ \text{conditions} \end{array}
\begin{cases} \delta_i^L + \delta_{i+1}^L \leq 1 \\ \delta_i^L + \delta_{i+2}^L \leq 1 \\ \quad \cdots \\ \delta_i^L + \delta_{i+(k_L-1)}^L \leq 1 \end{cases} \\
\text{If } i = S_L-(k_L-2), \\
\qquad \begin{array}{c} k_L-2 \\ \text{conditions} \end{array}
\begin{cases} \delta_i^L + \delta_{i+1}^L \leq 1 \\ \delta_i^L + \delta_{i+2}^L \leq 1 \\ \quad \cdots \\ \delta_i^L + \delta_{i+(k_L-2)}^L \leq 1 \end{cases} \\
\text{If } i = S_L-(k_L-3), \\
\qquad \begin{array}{c} k_L-3 \\ \text{conditions} \end{array}
\begin{cases} \delta_i^L + \delta_{i+1}^L \leq 1 \\ \delta_i^L + \delta_{i+2}^L \leq 1 \\ \quad \cdots \\ \delta_i^L + \delta_{i+(k_L-3)}^L \leq 1 \end{cases} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\text{If } i = S_L-\big(k_L-(k_L-1)\big) = S_L-1, \\
\qquad 1 \text{ condition} \ \ \big\{ \delta_i^L + \delta_{i+1}^L \leq 1
\end{array}
\right\} \qquad (11b)
$$

Let us consider ℓ non-interruptible loads $L_1, L_2, \dots, L_\ell$ which are connected to the home grid. The power consumption of these loads is $P_1^L, P_2^L, \dots, P_\ell^L$. The objective now is to determine the contribution of these non-interruptible loads to the total controllable loads consumption, and to find the optimal fashion to schedule these loads.

Let us take $q \in \mathbb{N}$. The parameter q allows us to refine the subdivision by dividing each time interval into q new intervals of same length $\frac{\tau}{q}$. In this chapter we will keep $q = 1$. We will discuss this parameter more in depth in chapter 6.

Define P_{nom}^{bt} the nominal power of the biofuel burning turbine.

Define $\forall j \in [\![1, N]\!]$, $\delta_j^{bt} \in [\![0,1]\!]$, with $\delta_j^{bt} = 0$ if the turbine is off over d_i, and $\delta_j^{bt} = 1$ if the turbine is on over d_i.

In the rest of this section, for the system of ℓ non-interruptible loads considered, we express the corresponding inequality conditions {(α), (β), (11b)} and equality conditions (11a) in matrix form.

4.2.2- Inequality conditions in matrix form

For a non-interruptible load L, let the matrices $\boldsymbol{M_{N,q}^{\alpha,L}}$, $\boldsymbol{M_{N,q}^{\beta,L}}$, $\boldsymbol{M_{S_L}^{k_L-j}}$ and $\boldsymbol{M_{S_L}}$ be:

$$
\boldsymbol{M_{N,q}^{\alpha,L}} = \underbrace{\begin{bmatrix}
1 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
2 & \vdots & \vdots & & & & \vdots \\
3 & 0 & \vdots & & & & \vdots \\
\vdots & 1 & 0 & & & & \vdots \\
k_L - 1 & 2 & 1 & & \ddots & & \vdots \\
k_L & 3 & 2 & & & & 0 \\
k_L & \vdots & 3 & & & & 1 \\
\vdots & k_L - 1 & \vdots & & & & 2 \\
\vdots & k_L & k_L - 1 & & & & 3 \\
\vdots & \vdots & k_L & & & & \vdots \\
\vdots & \vdots & \vdots & & & \ddots & k_L - 1 \\
k_L & k_L & k_L & \cdots & \cdots & k_L & k_L
\end{bmatrix}}_{S_L}
\begin{array}{l}
\\ q \\ 2q \\ (N-k_L)q \\ \\ \\ \\ \\ k_L \\ \\ \\ \\
\end{array}
$$

$$
\boldsymbol{M}_{\boldsymbol{N},\boldsymbol{q}}^{\boldsymbol{\beta},\boldsymbol{L}} = \underbrace{\begin{bmatrix}
1 & 0 & \dots & \dots & \dots & \dots & 0 \\
\vdots & \vdots & & & & & \vdots \\
1 & 0 & \dots & \dots & \dots & \dots & 0 \\
1 & 1 & 0 & \dots & \dots & \dots & 0 \\
\vdots & \vdots & & & & & \vdots \\
1 & 1 & 0 & \dots & \dots & \dots & 0 \\
1 & 0 & 1 & 0 & \dots & \dots & 0 \\
\vdots & \vdots & \vdots & & & & \vdots \\
1 & 0 & 1 & 0 & \dots & \dots & 0 \\
 & \dots & & \dots & & \dots & \\
1 & 0 & \dots & \dots & \dots & 0 & 1 \\
\vdots & & & & & \vdots & \vdots \\
1 & 0 & \dots & \dots & \dots & 0 & 1
\end{bmatrix}}_{S_L}
\begin{array}{l} \updownarrow q \\ \\ \\ \\ \end{array}
\quad N \times q
$$

$$
\boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{k_L-j}} = \overbrace{\begin{bmatrix}
1 & 1 & 0 & 0 & \dots & \dots & 0 \\
1 & 0 & 1 & 0 & \dots & \dots & \vdots \\
\vdots & & & & & & 0 \\
1 & 0 & \dots & \dots & \dots & 0 & 1
\end{bmatrix}}^{k_L - j} \quad k_L
$$

$$
\begin{array}{r} S_L - k_L + 1 \\ \\ \\ \end{array}
\boldsymbol{M}_{\boldsymbol{S_L}} = \underbrace{\begin{bmatrix}
\boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{k_L-1}} & \mathbf{0}_{\mathbb{R}^{k_L-1}} & \dots & \dots & \dots & \dots & & \mathbf{0}_{\mathbb{R}^{k_L-1}} \\
\mathbf{0}_{\mathbb{R}^{k_L-1}} & \boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{k_L-1}} & \mathbf{0}_{\mathbb{R}^{k_L-1}} & & & & & \vdots \\
\vdots & \ddots & \ddots & \ddots & & & & \vdots \\
\mathbf{0}_{\mathbb{R}^{k_L-1}} & \dots & \mathbf{0}_{\mathbb{R}^{k_L-1}} & \boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{k_L-1}} & \mathbf{0}_{\mathbb{R}^{k_L-1}} & \dots & \dots & \mathbf{0}_{\mathbb{R}^{k_L-1}} \\
\mathbf{0}_{\mathbb{R}^{k_L-2}} & & & & \boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{k_L-2}} & \mathbf{0}_{\mathbb{R}^{k_L-2}} & & \mathbf{0}_{\mathbb{R}^{k_L-2}} \\
\mathbf{0}_{\mathbb{R}^{k_L-3}} & & & & & \boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{k_L-3}} & & \mathbf{0}_{\mathbb{R}^{k_L-3}} \\
\vdots & & & & & & \ddots & \\
\mathbf{0}_{\mathbb{R}^{1}} & \dots & \dots & \dots & \dots & \dots & \mathbf{0}_{\mathbb{R}^{1}} & \boldsymbol{M}_{\boldsymbol{S_L}}^{\boldsymbol{1}}
\end{bmatrix}}_{S_L}
\quad (k_L - 1) \times (S_L - \frac{k_L}{2})
$$

$\boldsymbol{M}_{\boldsymbol{N},\boldsymbol{q}}^{\boldsymbol{\alpha},\boldsymbol{L}}$ is a $((N - k_L)q + k_L) \times S_L$ matrix and $\boldsymbol{M}_{\boldsymbol{N},\boldsymbol{q}}^{\boldsymbol{\beta},\boldsymbol{L}}$ is a $Nq \times S_L$ matrix.
$\boldsymbol{M}_{\boldsymbol{S_L}}$ is a $(k_L - 1)(S_L - \frac{k_L}{2}) \times S_L$ matrix.

Let also $\boldsymbol{T_N}$ and $\boldsymbol{I_N}$ be:

$$\boldsymbol{T_N} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 1 & 1 & \ddots & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 1 & 1 & & \ddots & 0 \\ 1 & 1 & \dots & \dots & 1 \end{bmatrix} \qquad \boldsymbol{I_N} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & \ddots & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & & \ddots & 0 \\ 0 & 0 & \dots & \dots & 1 \end{bmatrix}$$

$\boldsymbol{T_N}$ and $\boldsymbol{I_N}$ are $N \times N$ matrices.

Finally, define $\boldsymbol{A}^{(\boldsymbol{I})}$, $\boldsymbol{B}^{(\boldsymbol{I})}$and $\boldsymbol{X}^{(\boldsymbol{I})}$:

$$\boldsymbol{A}^{(I)} = \tau. \begin{bmatrix} -\boldsymbol{T_N} & -P_{nom}^{bt}\boldsymbol{T_N} & P_1^L.\boldsymbol{M_{N,1}^{\alpha,L_1}} & P_2^L.\boldsymbol{M_{N,1}^{\alpha,L_2}} & \dots & P_\ell^L.\boldsymbol{M_{N,1}^{\alpha,L_\ell}} \\ \boldsymbol{T_N} & P_{nom}^{bt}\boldsymbol{T_N} & -P_1^L.\boldsymbol{M_{N,1}^{\alpha,L_1}} & -P_2^L.\boldsymbol{M_{N,1}^{\alpha,L_2}} & \dots & -P_\ell^L.\boldsymbol{M_{N,1}^{\alpha,L_\ell}} \\ \boldsymbol{I_N} & P_{nom}^{bt}\boldsymbol{I_N} & -P_1^L.\boldsymbol{M_{N,1}^{\beta,L_1}} & -P_2^L.\boldsymbol{M_{N,1}^{\beta,L_2}} & \dots & -P_\ell^L.\boldsymbol{M_{N,1}^{\beta,L_\ell}} \\ -\boldsymbol{I_N} & -P_{nom}^{bt}\boldsymbol{I_N} & P_1^L.\boldsymbol{M_{N,1}^{\beta,L_1}} & P_2^L.\boldsymbol{M_{N,1}^{\beta,L_2}} & \dots & P_\ell^L.\boldsymbol{M_{N,1}^{\beta,L_\ell}} \\ & & \frac{1}{\tau}\boldsymbol{M_{S_{L_1}}} & & & \\ & & & \frac{1}{\tau}\boldsymbol{M_{S_{L_2}}} & & \\ & & & & \ddots & \\ & & & & & \frac{1}{\tau}\boldsymbol{M_{S_{L_\ell}}} \end{bmatrix}$$

$$\boldsymbol{B}^{(I)} = \begin{bmatrix} \boldsymbol{b}_{\boldsymbol{\alpha},\mathbf{1}}^{(I)} \\ \boldsymbol{b}_{\boldsymbol{\alpha},\mathbf{2}}^{(I)} \\ \boldsymbol{b}_{\boldsymbol{\beta},\mathbf{1}}^{(I)} \\ \boldsymbol{b}_{\boldsymbol{\beta},\mathbf{2}}^{(I)} \\ \mathbf{1}_{\mathbb{R}^{\sum_{i=1}^{\ell}\left(k_{L_i}-1\right)\times\left(S_{L_i}-\frac{k_{L_i}}{2}\right)}} \end{bmatrix} \qquad \boldsymbol{X}^{(I)} = \begin{bmatrix} P_1^{(g)} \\ \vdots \\ P_N^{(g)} \\ \delta_1^{bt} \\ \vdots \\ \delta_N^{bt} \\ \delta_1^{L_1} \\ \vdots \\ \delta_{S_{L_1}}^{L_1} \\ \vdots \\ \vdots \\ \delta_1^{L_\ell} \\ \vdots \\ \delta_{S_{L_\ell}}^{L_\ell} \end{bmatrix}$$

With:

$$
\boldsymbol{b}_{\boldsymbol{\alpha},\mathbf{1}}^{(\boldsymbol{I})} = \begin{bmatrix} E^{init} + (P_1^{(s)} + P_1^{(w)}).\tau - P_1^{(ncl)}.\tau \\ \vdots \\ E^{init} + \sum_{i=1}^{1<n<N} (P_i^{(s)} + P_i^{(w)}).\tau - \sum_{i=1}^{1<n<N} P_i^{(ncl)}.\tau \\ \vdots \\ E^{init} + \sum_{i=1}^{N} (P_i^{(s)} + P_i^{(w)}).\tau - \sum_{i=1}^{N} P_i^{(ncl)}.\tau \end{bmatrix}
$$

$$
\boldsymbol{b}_{\boldsymbol{\alpha},\mathbf{2}}^{(\boldsymbol{I})} = \begin{bmatrix} E^{max} - E^{init} - \left(P_1^{(s)} + P_1^{(w)}\right).\tau + P_1^{(ncl)}.\tau \\ \vdots \\ E^{max} - E^{init} - \sum_{i=1}^{1<n<N} (P_i^{(s)} + P_i^{(w)}).\tau + \sum_{i=1}^{1<n<N} P_i^{(ncl)}.\tau \\ \vdots \\ E^{max} - E^{init} - \sum_{i=1}^{N} (P_i^{(s)} + P_i^{(w)}).\tau + \sum_{i=1}^{N} P_i^{(ncl)}.\tau \end{bmatrix}
$$

$$
\boldsymbol{b}_{\boldsymbol{\beta},\mathbf{1}}^{(\boldsymbol{I})} = \begin{bmatrix} R^c - \left(P_1^{(s)} + P_1^{(w)}\right).\tau + P_1^{(ncl)}.\tau \\ \vdots \\ R^c - \left(P_i^{(s)} + P_i^{(w)}\right).\tau + P_i^{(ncl)}.\tau \\ \vdots \\ R^c - \left(P_N^{(s)} + P_N^{(w)}\right).\tau + P_N^{(ncl)}.\tau \end{bmatrix} \qquad \boldsymbol{b}_{\boldsymbol{\beta},\mathbf{2}}^{(\boldsymbol{I})} = \begin{bmatrix} R^d + (P_1^{(s)} + P_1^{(w)}).\tau - P_1^{(ncl)}.\tau \\ \vdots \\ R^d + (P_i^{(s)} + P_i^{(w)}).\tau - P_i^{(ncl)}.\tau \\ \vdots \\ R^d + (P_N^{(s)} + P_N^{(w)}).\tau - P_N^{(ncl)}.\tau \end{bmatrix}
$$

Then:

$$
\boxed{\{(\alpha), (\beta), (11b)\} \text{ is equivalent to } \boldsymbol{A}^{(\boldsymbol{I})}\boldsymbol{X}^{(\boldsymbol{I})} \leq \boldsymbol{B}^{(\boldsymbol{I})}}
$$

4.2.3- <u>Equality conditions in matrix form</u>

Let now $\boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{I})}$ and $\boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{I})}$ be:

$$
\boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{I})} = \begin{array}{c} \begin{array}{ccccc} \xleftrightarrow{\quad 2N \quad} & \xleftrightarrow{\quad S_{L_1} \quad} & \xleftrightarrow{\quad S_{L_2} \quad} & \qquad\qquad & \xleftrightarrow{\quad S_{L_\ell} \quad} \end{array} \\ \begin{bmatrix} 0 & \dots & 0 & 1 & \dots & 1 & 0 & \dots & 0 & \dots & \dots & \dots & 0 & \dots & 0 \\ 0 & & 0 & 0 & \dots & 0 & 1 & \dots & 1 & \ddots & \dots & \dots & 0 & & 0 \\ 0 & & 0 & 0 & & 0 & 0 & \dots & 0 & \ddots & \ddots & & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \ddots & \ddots & \ddots & 0 & & 0 \\ \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & & \ddots & \ddots & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & \dots & \dots & 0 & 1 & \dots & 1 \end{bmatrix} \end{array} \Big\updownarrow \ \ell
$$

$$\boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{I})} = \begin{bmatrix} \gamma^{L_1} \\ \gamma^{L_2} \\ \vdots \\ \vdots \\ \gamma^{L_\ell} \end{bmatrix} \updownarrow \ell$$

Then:

$$\boxed{\text{(11a) is equivalent to } \boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{I})}\boldsymbol{X}^{(\boldsymbol{I})} = \boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{I})}}$$

4.2.4- <u>Equivalent system in matrix form</u>

Define $\boldsymbol{\mathcal{P}^u} = [p_1^u \quad p_2^u \quad \dots \quad p_{N-1}^u \quad p_N^u]^T$ the electricity prices corresponding to the subdivision d.

Define $\boldsymbol{\mathcal{P}^f} = p^f \mathbf{1}_{\mathbb{R}^\mathbf{N}}{}^T$ the biofuel prices corresponding to the subdivision d. Biofuel prices are supposed constant over D and equal to p^f.

Finally define $\widetilde{\boldsymbol{\mathcal{P}}}^{(I)} = [\boldsymbol{\mathcal{P}^u}^T \quad \boldsymbol{\mathcal{P}^f}^T \quad \underbrace{0 \quad \dots \quad 0}_{S_{L_1}+\dots+S_{L_\ell}}]^T$

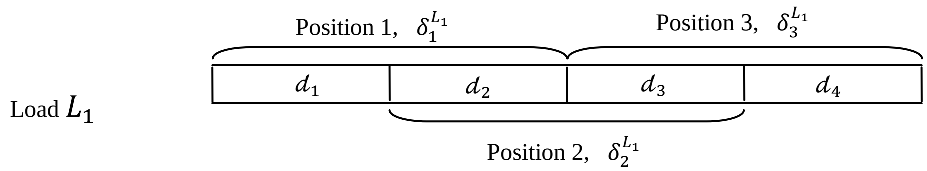
The non-interruptible loads optimal schedule problem can then be expressed as the following mixed-integer linear programming problem:

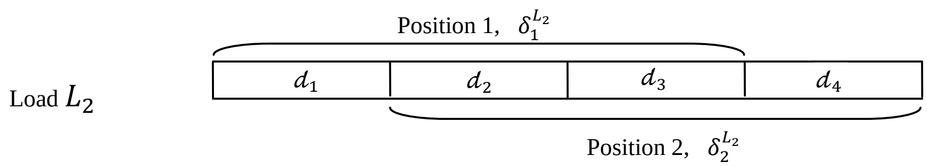
Minimize	$\mathcal{C}^{(I)} = \widetilde{\boldsymbol{\mathcal{P}}}^{(I)^T}.\boldsymbol{X}^{(\boldsymbol{I})}$	
Subject to	$\boldsymbol{A}^{(\boldsymbol{I})}\boldsymbol{X}^{(\boldsymbol{I})} \leq \boldsymbol{B}^{(\boldsymbol{I})}$ $\boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{I})}\boldsymbol{X}^{(\boldsymbol{I})} = \boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{I})}$	(LP-I)
	$\forall i \in [\![1, N]\!],\ \delta_i^{bt} \in [\![0,1]\!]$ $\forall i \in [\![1, \ell]\!], \forall j \in [\![1, S_{L_i}]\!],\ \delta_j^{L_i} \in [\![0,1]\!]$	

4.3 - Example

<u>Example</u> : for $N = 4$, $\ell = 2$, $k_{L_1} = 2$, $k_{L_2} = 3$, $\gamma^{L_1} = 1$, $\gamma^{L_2} = 1$

$S_{L_1} = N - k_{L_1} + 1 = 3, \quad S_{L_2} = N - k_{L_2} + 1 = 2$





$$\boldsymbol{M}_{N,1}^{\alpha,L_1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \\ 2 & 2 & 2 \end{bmatrix} \qquad \boldsymbol{M}_{N,1}^{\beta,L_1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{M}_{N,1}^{\alpha,L_2} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \\ 3 & 3 \end{bmatrix} \qquad \boldsymbol{M}_{N,1}^{\beta,L_2} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{M}_{S_{L_1}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \qquad \boldsymbol{M}_{S_{L_2}} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$
\boldsymbol{A}^{(I)} = \tau.\begin{bmatrix}
-1 & 0 & 0 & 0 & -P_{nom}^{bt} & 0 & 0 & 0 & P_1^L & 0 & 0 & P_2^L & 0 \\
-1 & -1 & 0 & 0 & -P_{nom}^{bt} & -P_{nom}^{bt} & 0 & 0 & 2P_1^L & P_1^L & 0 & 2P_2^L & P_2^L \\
-1 & -1 & -1 & 0 & -P_{nom}^{bt} & -P_{nom}^{bt} & -P_{nom}^{bt} & 0 & 2P_1^L & 2P_1^L & P_1^L & 3P_2^L & 2P_2^L \\
-1 & -1 & -1 & -1 & -P_{nom}^{bt} & -P_{nom}^{bt} & -P_{nom}^{bt} & -P_{nom}^{bt} & 2P_1^L & 2P_1^L & 2P_1^L & 3P_2^L & 3P_2^L \\
1 & 0 & 0 & 0 & P_{nom}^{bt} & 0 & 0 & 0 & -P_1^L & 0 & 0 & -P_2^L & 0 \\
1 & 1 & 0 & 0 & P_{nom}^{bt} & P_{nom}^{bt} & 0 & 0 & -2P_1^L & -P_1^L & 0 & -2P_2^L & -P_2^L \\
1 & 1 & 1 & 0 & P_{nom}^{bt} & P_{nom}^{bt} & P_{nom}^{bt} & 0 & -2P_1^L & -2P_1^L & -P_1^L & -3P_2^L & -2P_2^L \\
1 & 1 & 1 & 1 & P_{nom}^{bt} & P_{nom}^{bt} & P_{nom}^{bt} & P_{nom}^{bt} & -2P_1^L & -2P_1^L & -2P_1^L & -3P_2^L & -3P_2^L \\
1 & 0 & 0 & 0 & P_{nom}^{bt} & 0 & 0 & 0 & -P_1^L & 0 & 0 & -P_2^L & 0 \\
0 & 1 & 0 & 0 & 0 & P_{nom}^{bt} & 0 & 0 & -P_1^L & -P_1^L & 0 & -P_2^L & -P_2^L \\
0 & 0 & 1 & 0 & 0 & 0 & P_{nom}^{bt} & 0 & 0 & -P_1^L & -P_1^L & -P_2^L & -P_2^L \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & P_{nom}^{bt} & 0 & 0 & -P_1^L & 0 & -P_2^L \\
-1 & 0 & 0 & 0 & -P_{nom}^{bt} & 0 & 0 & 0 & P_1^L & 0 & 0 & P_2^L & 0 \\
0 & -1 & 0 & 0 & 0 & -P_{nom}^{bt} & 0 & 0 & P_1^L & P_1^L & 0 & P_2^L & P_2^L \\
0 & 0 & -1 & 0 & 0 & 0 & -P_{nom}^{bt} & 0 & 0 & P_1^L & P_1^L & P_2^L & P_2^L \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -P_{nom}^{bt} & 0 & 0 & P_1^L & 0 & P_2^L \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} & \frac{1}{\tau} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} & \frac{1}{\tau} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} & \frac{1}{\tau}
\end{bmatrix}
$$

$$
\boldsymbol{B}^{(\boldsymbol{I})} = \begin{bmatrix}
E^{init} + \left(P_1^{(s)} + P_1^{(w)}\right).\tau - P_1^{(ncl)}.\tau \\
\vdots \\
E^{init} + \sum_{i=1}^{4}\left(P_i^{(s)} + P_i^{(w)}\right).\tau - \sum_{i=1}^{4} P_i^{(ncl)}.\tau \\
E^{max} - E^{init} - \left(P_1^{(s)} + P_1^{(w)}\right).\tau + P_1^{(ncl)}.\tau \\
\vdots \\
E^{max} - E^{init} - \sum_{i=1}^{4}\left(P_i^{(s)} + P_i^{(w)}\right).\tau + \sum_{i=1}^{4} P_i^{(ncl)}.\tau \\
R^c - \left(P_1^{(s)} + P_1^{(w)}\right).\tau + P_1^{(ncl)}.\tau \\
\vdots \\
R^c - \left(P_4^{(s)} + P_4^{(w)}\right).\tau + P_4^{(ncl)}.\tau \\
R^d + \left(P_1^{(s)} + P_1^{(w)}\right).\tau - P_1^{(ncl)}.\tau \\
\vdots \\
R^d + \left(P_4^{(s)} + P_4^{(w)}\right).\tau - P_4^{(ncl)}.\tau
\end{bmatrix}
\qquad
\boldsymbol{X}^{(\boldsymbol{I})} = \begin{bmatrix}
P_1^{(g)} \\ P_2^{(g)} \\ P_3^{(g)} \\ P_4^{(g)} \\ \delta_1^{bt} \\ \delta_2^{bt} \\ \delta_3^{bt} \\ \delta_4^{bt} \\ \delta_1^{L_1} \\ \delta_2^{L_1} \\ \delta_3^{L_1} \\ \delta_1^{L_2} \\ \delta_2^{L_2}
\end{bmatrix}
$$

$$
\boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{I})} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1
\end{bmatrix}
\qquad
\boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{I})} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
$$

$\boldsymbol{A}^{(\boldsymbol{I})}\boldsymbol{X}^{(\boldsymbol{I})} \leq \boldsymbol{B}^{(\boldsymbol{I})}$ gives :

$$\left.\begin{array}{l}
(-P_1^{(g)} - \delta_1^{bt} P_{nom}^{bt} + \delta_1^{L_1} P_1^L + \delta_1^{L_2} P_2^L).\tau \leq E^{init} + \left(P_1^{(s)} + P_1^{(w)}\right).\tau - P_1^{(ncl)}.\tau \\[2ex]
\left(-\left(P_1^{(g)} + P_2^{(g)}\right) - \left(\delta_1^{bt} + \delta_2^{bt}\right) P_{nom}^{bt} + 2\delta_1^{L_1} P_1^L + \delta_2^{L_1} P_1^L + 2\delta_1^{L_2} P_2^L + \delta_2^{L_2} P_2^L\right).\tau \\
\qquad \leq E^{init} + \sum_{i=1}^{2} \left(P_i^{(s)} + P_i^{(w)}\right).\tau - \sum_{i=1}^{2} P_i^{(ncl)}.\tau \\[2ex]
\left(-\left(P_1^{(g)} + P_2^{(g)} + P_3^{(g)}\right) - \left(\delta_1^{bt} + \delta_2^{bt} + \delta_3^{bt}\right) P_{nom}^{bt} + 2\delta_1^{L_1} P_1^L + 2\delta_2^{L_1} P_1^L + \delta_3^{L_1} P_1^L + 3\delta_1^{L_2} P_2^L + 2\delta_2^{L_2} P_2^L\right).\tau \\
\qquad \leq E^{init} + \sum_{i=1}^{3} \left(P_i^{(s)} + P_i^{(w)}\right).\tau - \sum_{i=1}^{3} P_i^{(ncl)}.\tau \\[2ex]
\left(-\left(P_1^{(g)} + P_2^{(g)} + P_3^{(g)} + P_4^{(g)}\right) - \left(\delta_1^{bt} + \delta_2^{bt} + \delta_3^{bt} + \delta_4^{bt}\right) P_{nom}^{bt} + 2\delta_1^{L_1} P_1^L + 2\delta_2^{L_1} P_1^L + 2\delta_3^{L_1} P_1^L + 3\delta_1^{L_2} P_2^L \right. \\
\qquad \left. + 3\delta_2^{L_2} P_2^L\right).\tau \leq E^{init} + \sum_{i=1}^{4} \left(P_i^{(s)} + P_i^{(w)}\right).\tau - \sum_{i=1}^{4} P_i^{(ncl)}.\tau
\end{array}\right\} \text{cf (7)}$$

$$\left.\begin{array}{l}
(P_1^{(g)} + \delta_1^{bt} P_{nom}^{bt} - \delta_1^{L_1} P_1^L - \delta_1^{L_2} P_2^L).\tau \leq E^{max} - E^{init} - \left(P_1^{(s)} + P_1^{(w)}\right).\tau + P_1^{(ncl)}.\tau \\[2ex]
\left(\left(P_1^{(g)} + P_2^{(g)}\right) + \left(\delta_1^{bt} + \delta_2^{bt}\right) P_{nom}^{bt} - 2\delta_1^{L_1} P_1^L - \delta_2^{L_1} P_1^L - 2\delta_1^{L_2} P_2^L - \delta_2^{L_2} P_2^L\right). \\
\qquad \leq E^{max} - E^{init} - \sum_{i=1}^{2} \left(P_i^{(s)} + P_i^{(w)}\right).\tau + \sum_{i=1}^{2} P_i^{(ncl)}.\tau \\[2ex]
\left(\left(P_1^{(g)} + P_2^{(g)} + P_3^{(g)}\right) + \left(\delta_1^{bt} + \delta_2^{bt} + \delta_3^{bt}\right) P_{nom}^{bt} - 2\delta_1^{L_1} P_1^L - 2\delta_2^{L_1} P_1^L - \delta_3^{L_1} P_1^L - 3\delta_1^{L_2} P_2^L - 2\delta_2^{L_2} P_2^L\right).\tau \\
\qquad \leq E^{max} - E^{init} - \sum_{i=1}^{3} \left(P_i^{(s)} + P_i^{(w)}\right).\tau + \sum_{i=1}^{3} P_i^{(ncl)}.\tau \\[2ex]
\left(\left(P_1^{(g)} + P_2^{(g)} + P_3^{(g)} + P_4^{(g)}\right) + \left(\delta_1^{bt} + \delta_2^{bt} + \delta_3^{bt} + \delta_4^{bt}\right) P_{nom}^{bt} - 2\delta_1^{L_1} P_1^L - 2\delta_2^{L_1} P_1^L - 2\delta_3^{L_1} P_1^L - 3\delta_1^{L_2} P_2^L \right. \\
\qquad \left. - 3\delta_2^{L_2} P_2^L\right).\tau \leq E^{max} - E^{init} - \sum_{i=1}^{4} \left(P_i^{(s)} + P_i^{(w)}\right).\tau + \sum_{i=1}^{4} P_i^{(ncl)}.\tau
\end{array}\right\} \text{cf (8)}$$

$$\left.\begin{array}{l}
(P_1^{(g)} + \delta_1^{bt} P_{nom}^{bt} - \delta_1^{L_1} P_1^L - \delta_1^{L_2} P_2^L).\tau \leq R^c - \left(P_1^{(s)} + P_1^{(w)}\right).\tau + P_1^{(ncl)}.\tau \\[2ex]
\left(P_2^{(g)} + \delta_2^{bt} P_{nom}^{bt} - \delta_1^{L_1} P_1^L - \delta_2^{L_1} P_1^L - \delta_1^{L_2} P_2^L - \delta_2^{L_2} P_2^L\right).\tau \leq R^c - \left(P_2^{(s)} + P_2^{(w)}\right).\tau + P_2^{(ncl)}.\tau \\[2ex]
\left(P_3^{(g)} + \delta_3^{bt} P_{nom}^{bt} - \delta_2^{L_1} P_1^L - \delta_3^{L_1} P_1^L - \delta_1^{L_2} P_2^L - \delta_2^{L_2} P_2^L\right).\tau \leq R^c - \left(P_3^{(s)} + P_3^{(w)}\right).\tau + P_3^{(ncl)}.\tau \\[2ex]
\left(P_4^{(g)} + \delta_4^{bt} P_{nom}^{bt} - \delta_3^{L_1} P_1^L - \delta_2^{L_2} P_2^L\right).\tau \leq R^c - \left(P_4^{(s)} + P_4^{(w)}\right).\tau + P_4^{(ncl)}.\tau
\end{array}\right\} \text{cf (9)}$$

$$\left.\begin{array}{l}
(-P_1^{(g)} - \delta_1^{bt} P_{nom}^{bt} + \delta_1^{L_1} P_1^L + \delta_1^{L_2} P_2^L).\tau \leq R^d + \left(P_1^{(s)} + P_1^{(w)}\right).\tau - P_1^{(ncl)}.\tau \\[2ex]
\left(-P_2^{(g)} - \delta_2^{bt} P_{nom}^{bt} + \delta_1^{L_1} P_1^L + \delta_2^{L_1} P_1^L + \delta_1^{L_2} P_2^L + \delta_2^{L_2} P_2^L\right).\tau \leq R^d + \left(P_2^{(s)} + P_2^{(w)}\right).\tau - P_2^{(ncl)}.\tau \\[2ex]
\left(-P_3^{(g)} - \delta_3^{bt} P_{nom}^{bt} + \delta_2^{L_1} P_1^L + \delta_3^{L_1} P_1^L + \delta_1^{L_2} P_2^L + \delta_2^{L_2} P_2^L\right).\tau \leq R^d + \left(P_3^{(s)} + P_3^{(w)}\right).\tau - P_3^{(ncl)}.\tau \\[2ex]
\left(-P_4^{(g)} - \delta_4^{bt} P_{nom}^{bt} + \delta_3^{L_1} P_1^L + \delta_2^{L_2} P_2^L\right).\tau \leq R^d + \left(P_4^{(s)} + P_4^{(w)}\right).\tau - P_4^{(ncl)}.\tau
\end{array}\right\} \text{cf (10)}$$

$$\left.\begin{array}{l}
\delta_1^{L_1} + \delta_2^{L_1} \leq 1 \\
\delta_2^{L_1} + \delta_3^{L_1} \leq 1 \\
\delta_1^{L_2} + \delta_2^{L_2} \leq 1
\end{array}\right\} \text{cf (11b)}$$

$\boldsymbol{Aeq}^{(\boldsymbol{I})}\boldsymbol{X}^{(\boldsymbol{I})} = \boldsymbol{Beq}^{(\boldsymbol{I})}$ gives :

$$\left.\begin{array}{l}
\delta_1^{L_1} + \delta_2^{L_1} + \delta_3^{L_1} = 1 \\
\delta_1^{L_2} + \delta_2^{L_2} = 1
\end{array}\right\} \text{cf (11a)}$$

4.4 - Simulations

4.4.1- Introduction

To conclude this chapter, we simulate successively five scenarios in order to illustrate the mathematical theory discussed above. For each scenario, the system (LP-I) is solved for the home grid defined in Table 1.

TABLE 1
HOMEGRID PARAMETERS – CHAPTER 4

Symbol	Quantity	Assumed value
D	Time horizon	24h
N	Number of time intervals	48
τ	Length of each time interval	30min
p^f	Biofuel price	7.4 ¢/kWh
E^{init}	Energy stored at $t=0$	0 kWh
E^{max}	Maximum capacity - storage device	3 kWh
R^c	Maximal charging rate - storage device	0.5 kWh
R^d	Maximal discharging rate - storage device	0.5 kWh
P_{nom}^{bt}	Nominal power - biofuel-burning turbine	1.5 kW
$\eta^{(s)}$	Solar panel efficiency	0.15
$S^{(s)}$	Solar panel eq. surface	1m^2
$\eta^{(w)}$	Wind turbine efficiency	0.20
$S^{(w)}$	Wind turbine eq. surface	1m^2
ℓ	Number of non-interruptible loads	6
P_1^L	Washing machine consumption	1 kW
P_2^L	Dryer consumption	1.5 kW
P_3^L	Dishwasher consumption	1 kW
P_4^L	Defrost cycle consumption	1.3 kW
P_5^L	Water heater consumption	2 kW
P_6^L	PHEV battery consumption	3 kW
k_{L_1}	$k_{L_1}\tau$ is the cycle of L_1	1
k_{L_2}	$k_{L_2}\tau$ is the cycle of L_2	2
k_{L_3}	$k_{L_3}\tau$ is the cycle of L_3	2
k_{L_4}	$k_{L_4}\tau$ is the cycle of L_4	1
k_{L_5}	$k_{L_5}\tau$ is the cycle of L_5	1
k_{L_6}	$k_{L_6}\tau$ is the cycle of L_6	1
γ^{L_1}	L_1 to be scheduled γ^{L_1} times	1
γ^{L_2}	L_2 to be scheduled γ^{L_2} times	1
γ^{L_3}	L_3 to be scheduled γ^{L_3} times	1
γ^{L_4}	L_4 to be scheduled γ^{L_4} times	1
γ^{L_5}	L_5 to be scheduled γ^{L_5} times	1
γ^{L_6}	L_6 to be scheduled γ^{L_6} times	4

Note that there are $\prod_{i=1}^{\ell}\binom{S_{L_i}}{\gamma^{L_i}} = 4.75 \times 10^{13}$ potential solutions in the 6-dimensionnal solution space for the non-interruptible loads schedule.

The day-ahead price signal is broadcasted to the smart meter and the non-controllable consumption forecasts are determined by a corresponding forecasting module which is out of the scope of this thesis.

It is assumed for this simulation that a maximum of 7kW can be exchanged at any time between the utility grid and the home grid.

The price signal and the non-controllable loads consumption forecasts assumed in this chapter are shown in Figure 3.

In scenario 1 to 5, we progressively enable the devices connected to the home grid and examine their impact on the electricity bill. The successive results are summarized at the end of this chapter in Table 2.

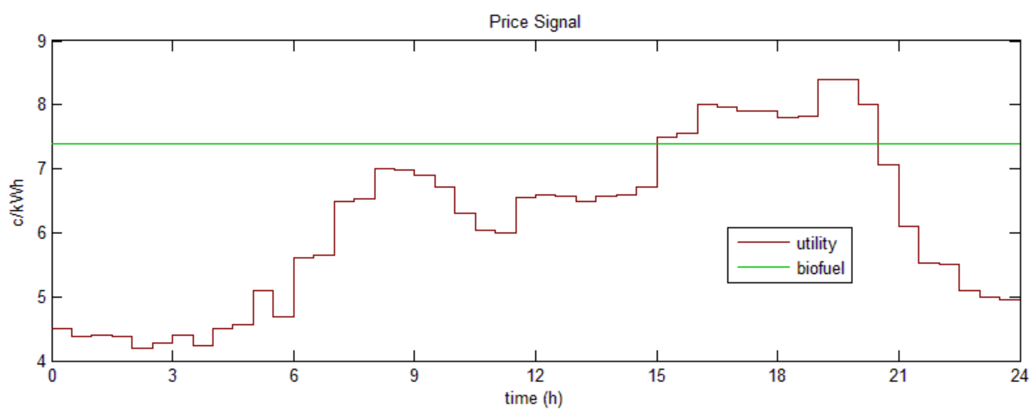


FIGURE 3: Price forecasts for scenarios 1 to 5

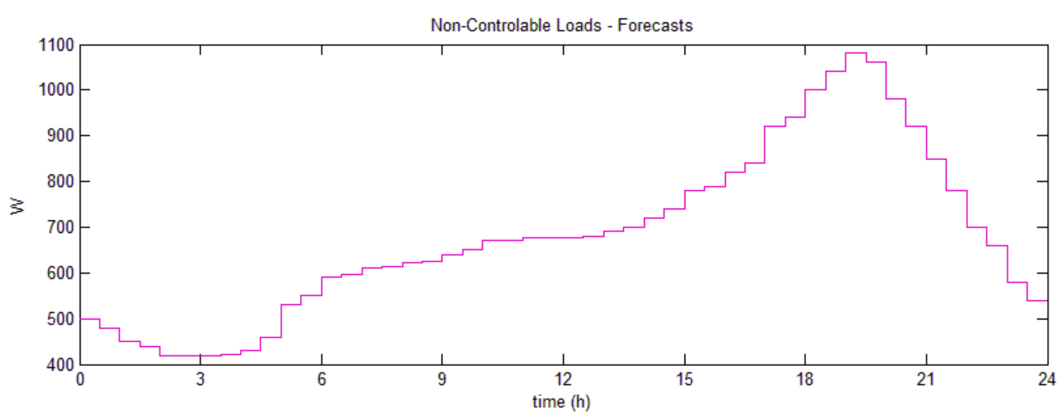


FIGURE 4: Non-controllable loads forecasts for scenarios 1 to 5

4.4.2- Scenario 1

In this scenario, we assume that the renewable energy units (solar, wind, biofuel-burning turbine) and the storage system are all off-line. No time constraints apply to the controllable loads.

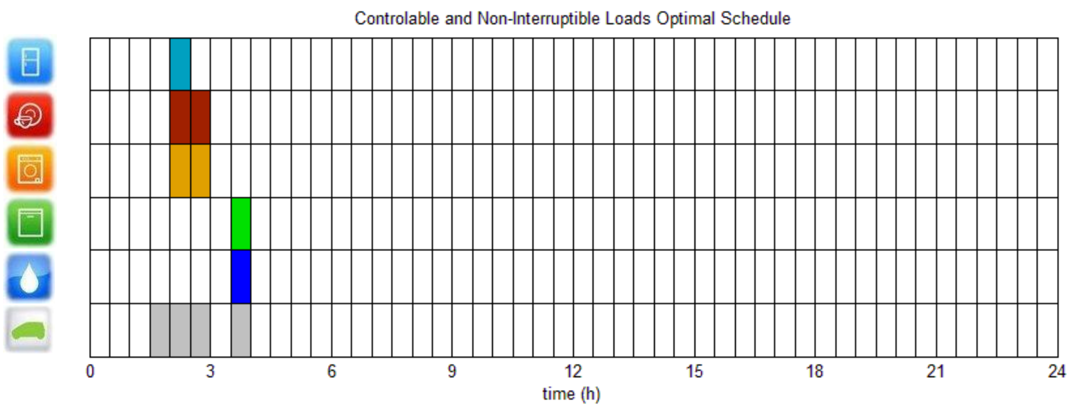


FIGURE 5: Optimal loads schedule - scenario 1

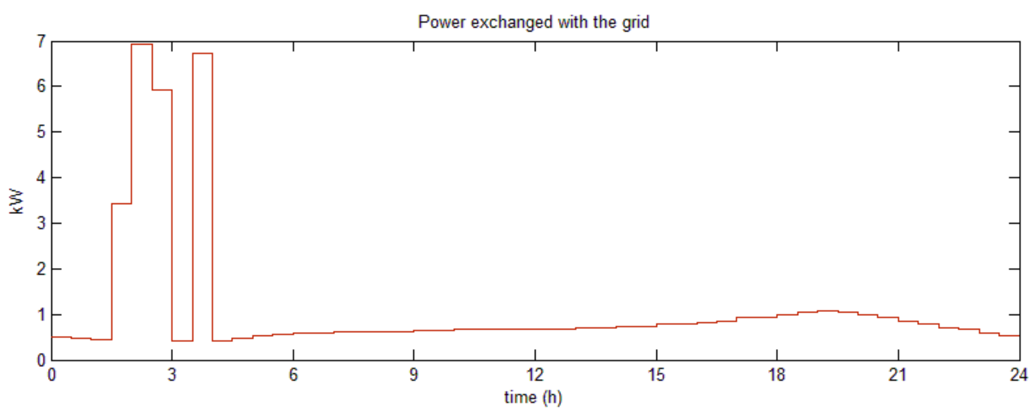


FIGURE 6: Power exchanged with the grid - scenario 1

The optimal schedule for this scenario corresponds to a forecasted cost of $1.51 for the next day. The controller obviously schedules all the loads between 1am and 4am since this time period corresponds to the lowest electricity prices for the next 24 hours. Notice that the 7 kW limit is enforced.

4.4.3- <u>Scenario 2</u>

In this scenario, time constraints apply to several controllable loads, and are transposed into constraints on the $\delta_i^{L_j}$:

- Loads 1 and 2 must run between 10am and 5pm
- Load 3 must run between 3pm and 7pm
- Load 6 must run between 0am and 6am

The purple areas in the loads schedule (Fig. 7) correspond to the non-scheduling periods.

The renewable energy units and the storage systems are still off-line.

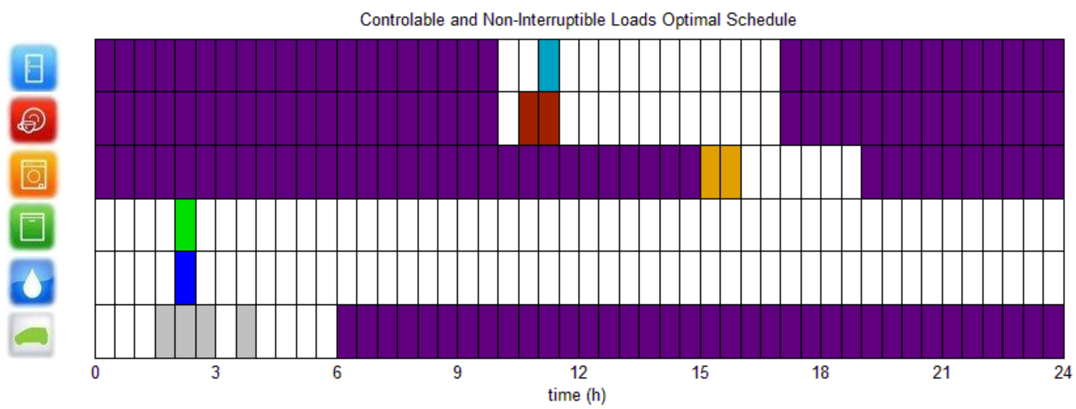


FIGURE 7: Optimal loads schedule - scenario 2

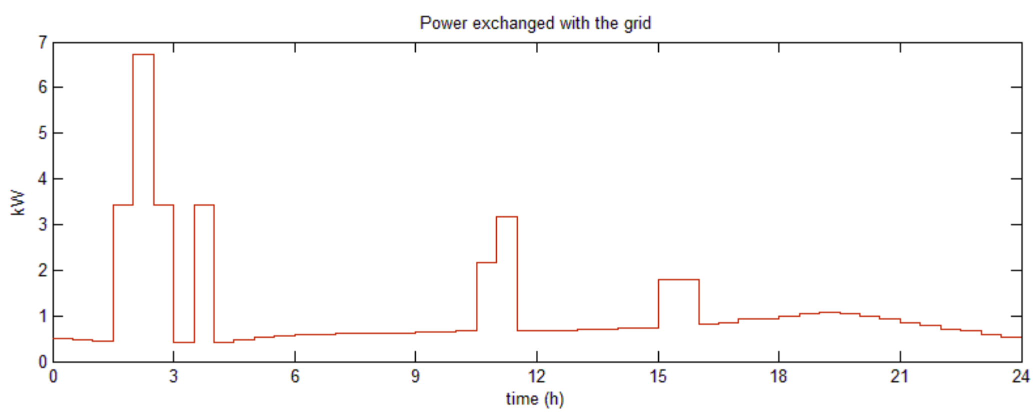


FIGURE 8: Power exchanged with the grid - scenario 2

The optimal schedule corresponds to a forecasted cost of $1.57 for the next day, which is obviously higher than the cost of a scenario not enforcing time constraints.

4.4.4- Scenario 3

In this scenario, the previous conditions are maintained, except that the solar and wind units are now online. Their production can be forecasted based on the irradiance and wind speed forecasts shown in Figures 9 and 10.

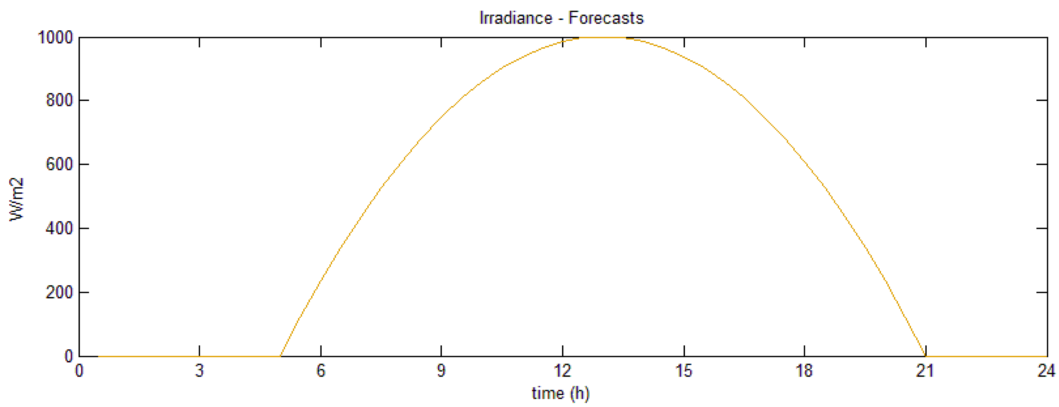


FIGURE 9: Irradiance forecasts - scenarios 3 to 5

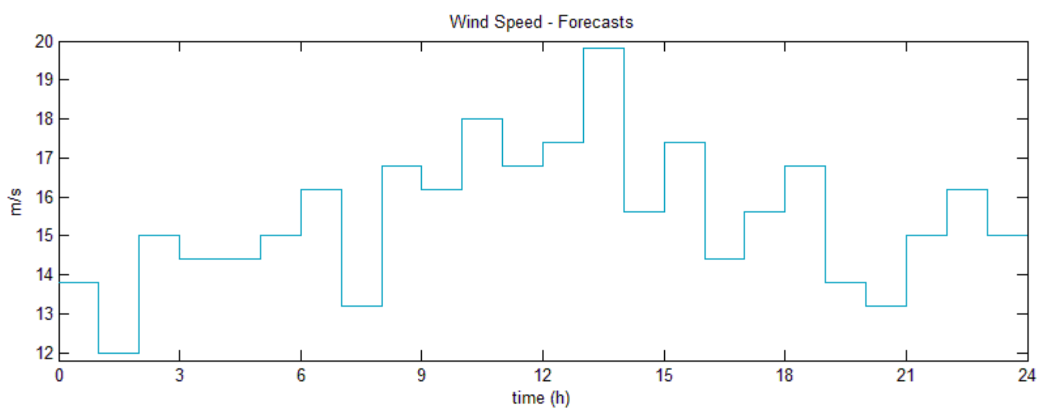


FIGURE 10: Wind speed forecasts - scenarios 3 to 5

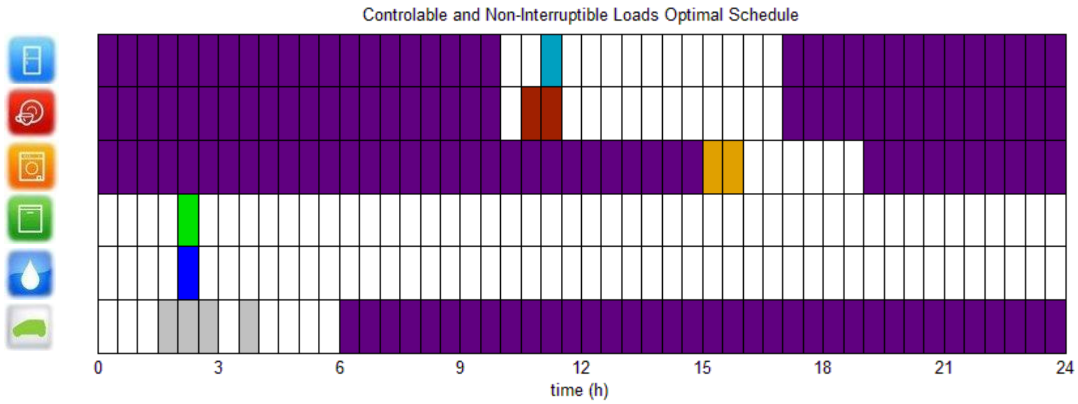


FIGURE 11: Optimal loads schedule - scenario 3

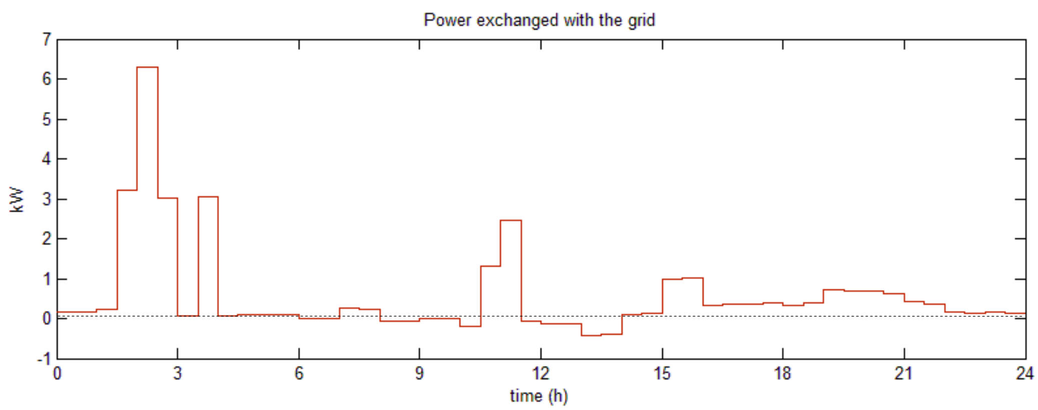


FIGURE 12: Power exchanged with the grid - scenario 3

The optimal schedule corresponds to a forecasted cost of $0.75 for the next day. The cost savings are due to the local power production. Notice that part of the local production is sold back to the utility grid, creating time periods where the resulting power flow exchanged between the utility grid and the home grid is negative.

4.4.5- Scenario 4

In this scenario, the previous conditions are maintained, except that the biofuel-burning turbine can now be turn on or off as needed.

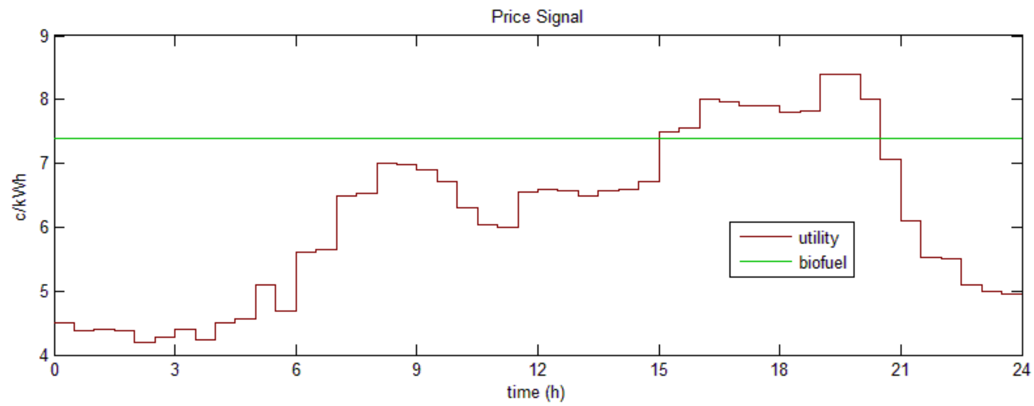


FIGURE 13: Price forecasts - scenario 4

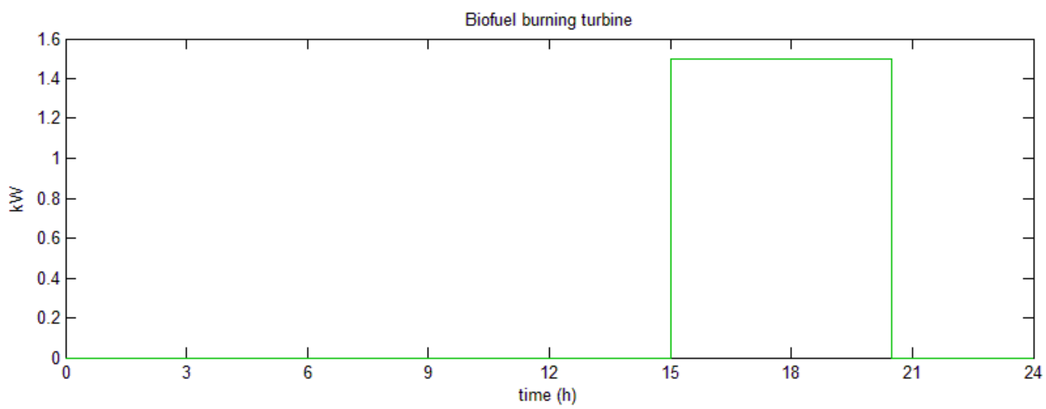


FIGURE 14: Biofuel-burning turbine schedule - scenario 4

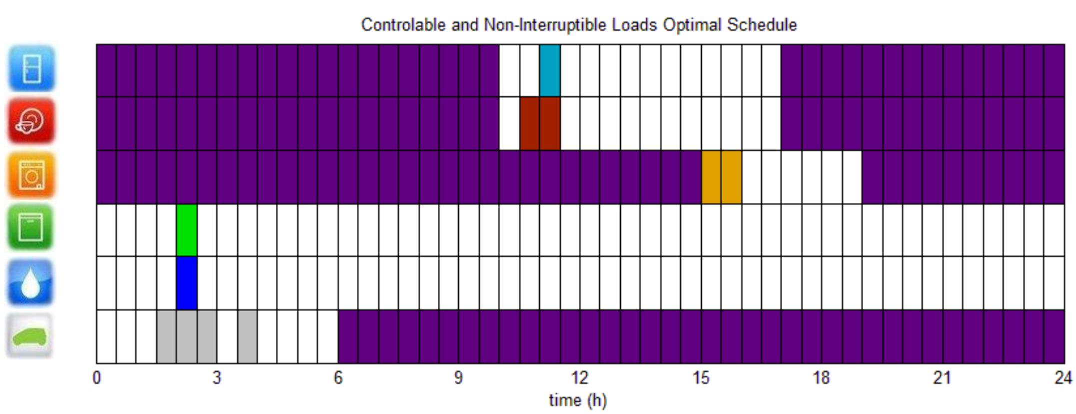


FIGURE 15: Optimal loads schedule - scenario 4

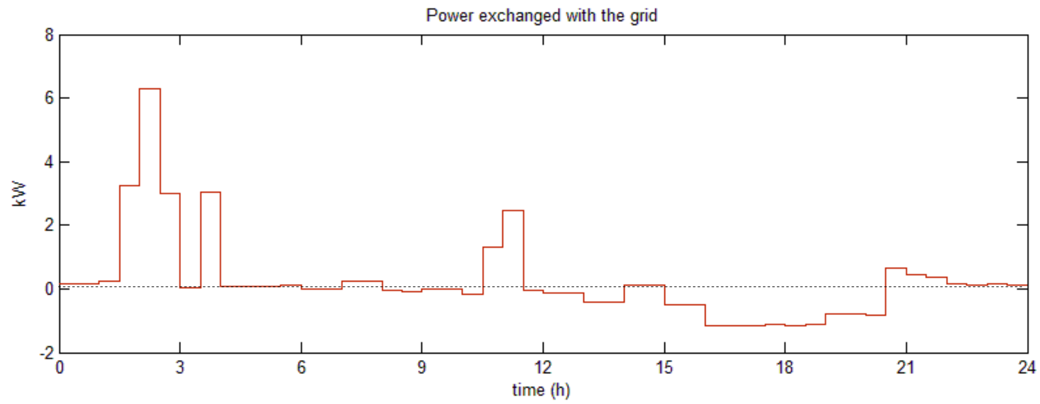


FIGURE 16: Power exchanged with the grid - scenario 4

The optimal schedule corresponds to a forecasted cost of $0.70 for the next day.

Notice that the biofuel-burning turbine starts as soon as the utility price signal goes above the biofuel price.

4.4.6- Scenario 5

In this scenario, the previous conditions are maintained, except that the storage system is now active.

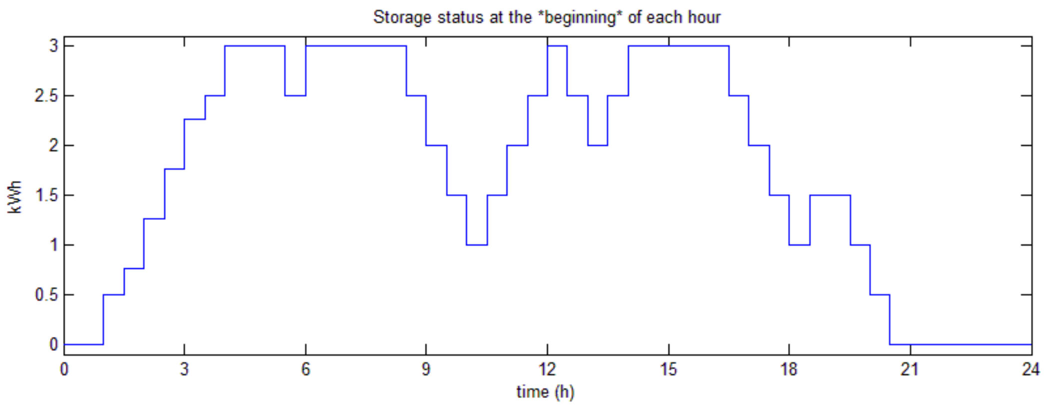


FIGURE 17: Storage schedule - scenario 4

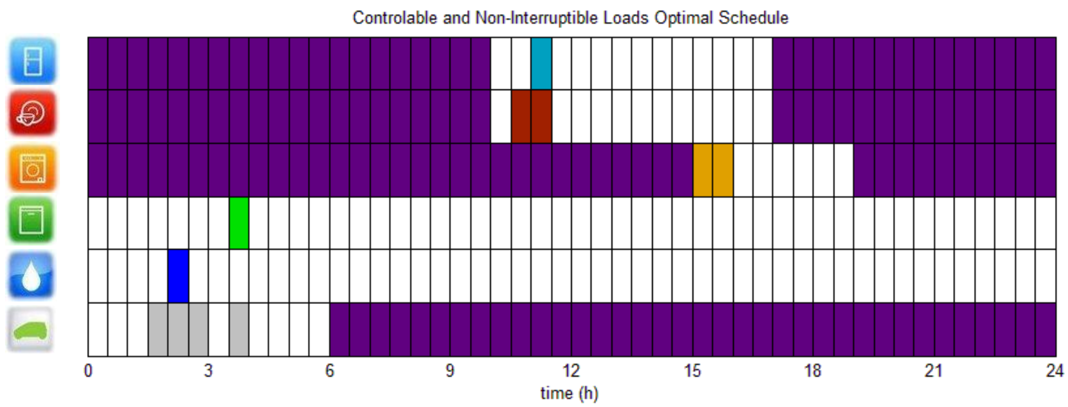


FIGURE 18: Optimal loads schedule - scenario 5

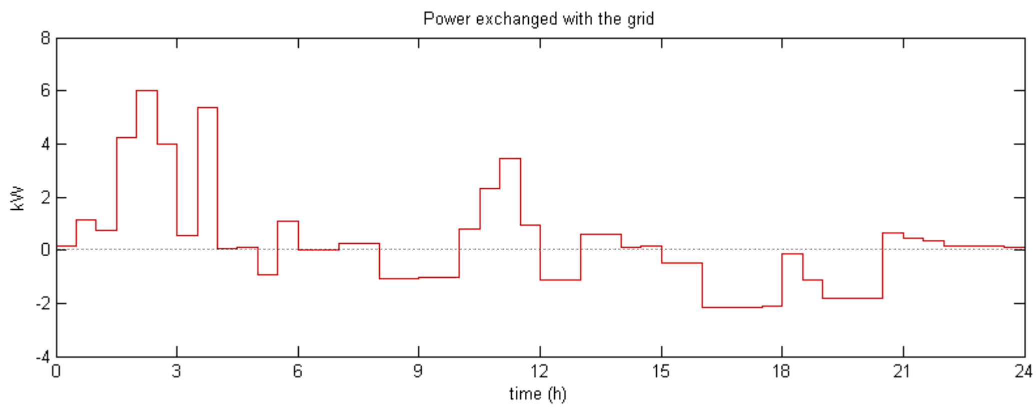


FIGURE 19: Power exchanged with the grid - scenario 5

The optimal schedule corresponds to a forecasted cost of $0.57 for the next day.

Notice that the storage device charges when prices are relatively low, and discharges when prices are relatively high.

Also notice that the storage device is completely discharged at the end of the 24 hour period as having energy stored in the device at the end would mean that the energy consumption is not optimized.

TABLE 2
SUMMARY OF THE SIMULATION RESULTS – CHAPTER 4

Scenario	Comfort preferences	Solar and wind units status	Biofuel-burning turbine status	Storage status	Cost ($)
1	No	Off-line	Off-line	Off-line	1.51
2	Yes	Off-line	Off-line	Off-line	1.57
3	Yes	Online	Off-line	Off-line	0.75
4	Yes	Online	Online	Off-line	0.70
5	Yes	Online	Online	Online	0.57

CHAPTER 5
THE INTERRUPTIBLE LOADS CASE

5.1 - Introduction

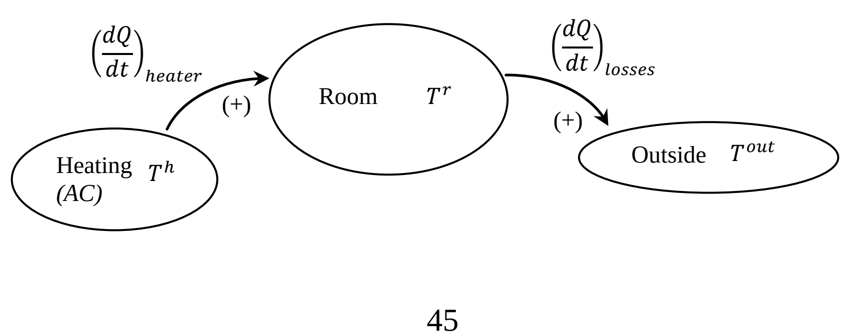
In this chapter, we focus on the heating and A/C systems which are interruptible loads, and assume that the non-interruptible loads are off-line. We also assume for now that the storage system and the generation capabilities are turned off. The non-controllable loads are not considered in order to simplify the derivation. Then, the utility grid is the only source providing power to the interruptible loads.

5.2 - Mathematical formulation as a linear programming problem

Let us consider the thermodynamic system modeling the house. This system exchanges heat with two other thermodynamic systems, the heating/AC, and the outside.

In winter, the heating system transfers heat to the house. As a result of this some heat is transferred to the outside world (thermal losses). In summer the heating system is replaced by the AC system and the heat flow is reversed.

In the following, we will use the term heating to refer to the system capable of producing heat in winter time, *and* cold in summer time.



The absolute temperature of the room T^r is the state variable which will allow us to describe the temporal evolution of the thermodynamic system throughout the next day.

T^r is a continuous function defined on a period of time $\mathcal{T}_{T^r} \subseteq \mathbb{R}^+$.

The user sets comfort preferences on T^r by defining two limit functions $T^{r,min}$ and $T^{r,max}$ defined on $\mathcal{T}_{T^r}$.

T^r must verify the following constraints:

$$\forall\ t \in \mathcal{T}_{T^r},\ T^{r,min}(t) \leq T^r(t) \leq T^{r,max}(t) \tag{12}$$

The evolution of the outside temperature T^{out} leads to variations in the quantity of heat $\left(\frac{dQ}{dt}\right)_{losses}$ exchanged between the outside and the room. This leads to variations in T^r. Thanks to the heating, heat is transferred to the room so that T^r can meet the constraints stated in (12) despite of external temperature variations.

For a given value of T^r, the user can control the value of $\left(\frac{dQ}{dt}\right)_{heater}$ through the temperature of the heater T^h, and then control the variations of T^r.

T^{out} and T^h are defined on $\mathcal{T}_{T^r}$. Additionally, physical constraints apply on the temperature of the heater : T^h must stay in $\left[T^{h,min},\ T^{h,max}\right] \subseteq \mathbb{R}$, and the variations of T^h are bounded by $R^{max} > 0$ and $R^{min} > 0$.

$$\forall\, t \in \mathcal{T}_{T^r}, \qquad T^{h,min} \leq T^h(t) \leq T^{h,max} \tag{13}$$

$$\forall\, t_1, t_2 \in \mathcal{T}_{T^r}, \qquad -R^{min} \leq T^h(t_2) - T^h(t_1) \leq R^{max} \tag{14}$$

The first law of thermodynamics applied to the thermodynamic system modeling the room gives $dU = \delta Q + \delta W$.

Since the process is isochoric, $dV = 0$, then $\delta W = -pdV = 0$. Hence $dU = \delta Q$.

The internal energy U is a state function, then $dU = \left(\frac{\partial U}{\partial T}\right)_P dT + \left(\frac{\partial U}{\partial P}\right)_T dP$, and $dP \simeq 0$ (the pressure does not vary significantly in the room). Hence $dU = \left(\frac{\partial U}{\partial T}\right)_P dT$.

Then, $dU = \delta Q = \left(\frac{\partial U}{\partial T}\right)_V dT = \mathcal{C} dT$, with $\mathcal{C}$ the heat capacity of air. And $\mathcal{C} = M_{air}.c$ with M_{air} the mass of air inside the house and c the specific heat capacity of air at constant pressure (heat capacity per unit mass).

Finally $\delta Q = M_{air}.cdT$ leads to :

$$\forall\, t \in \mathcal{T}_{T_r}, \quad \frac{dT^r(t)}{dt} = \frac{1}{M_{air}.c}\left\{\left(\frac{dQ(t)}{dt}\right)_{heater} - \left(\frac{dQ(t)}{dt}\right)_{losses}\right\} \tag{15}$$

From $\delta Q = M_{air}.cdT$ we can also derivate :

$$\forall\, t \in \mathcal{T}_{T_r}, \quad \left(\frac{dQ(t)}{dt}\right)_{heater} = \dot{M}.c \times \{T^h(t) - T^r(t)\} \tag{16}$$

$\dot{M}$ is the air flow rate through the heater and is assumed to be constant.

Let R_{eq} be the equivalent thermal resistance of the house.

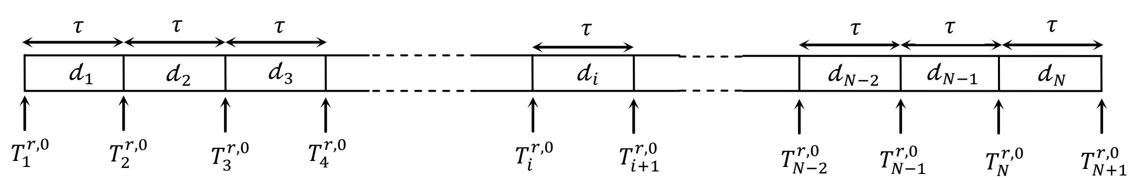
Refer to Appendix A for the geometry of the house, and Appendix B for the detailed calculation of R_{eq}.

We have :

$$\forall\, t \in \mathcal{T}_{T^r},\ \left(\frac{dQ(t)}{dt}\right)_{losses} = \frac{T^r(t) - T^{out}(t)}{R_{eq}} \quad (17)$$

Let now $d = \{d_1, d_2, \ldots, d_i, \ldots, d_N\}$ be a subdivision of D.

Define $T_i^{r,0}$ the temperature of the room at the beginning of the time interval d_i and T_i^r the function defined on d_i such that T_i^r and T^r are identically equal on d_i. The initial condition is given by $T^{init} = T_1^{r,0}$.



Assume that N is large enough so that $\tau \ll \tau_{T^{out}}$, with $\tau_{T^{out}}$ the characteristic time of T^{out}. Then we can assume that T^{out} is constant on d_i and equal to T_i^{out}.

Assume that T^h will be maintained constant on d_i and equal to T_i^h.

Assume that N is large enough so that $\tau \ll \tau_{T_i^r}$, with $\tau_{T_i^r}$ the characteristic time of T_i^r.

Then we can assume that :

$$\forall\, t \in d_i, \quad \left(\frac{dQ(t)}{dt}\right)_{heater} = \dot{M}.c \times \{T^h(t) - T^r(t)\} \simeq \dot{M}.c \times \{T_i^h - T_i^{r,0}\} \tag{18}$$

Define :

$$\forall\, i \in [\![1, N]\!], \quad P_i^h \triangleq \dot{M}.c \times \{T_i^h - T_i^{r,0}\} \tag{19}$$

When heat is transferred to the room (winter time), $P_i^h > 0$. When the heat flow is reversed (summer time, the "heater" behaves like an AC), $P_i^h < 0$.

Define $P_i^{h,+} > 0$ and $P_i^{h,-} > 0$ such that $P_i^h = P_i^{h,+} - P_i^{h,-}$.

Let us call $\eta^h > 0$ and $\eta^{ac} > 0$ the energy conversion efficiency of the heating and the A/C. Then, providing $P_i^{h,+}$ to the room requires $\eta^h \times P_i^{h,+}$ electric energy, and extracting $P_i^{h,-}$ from the room requires $\eta^{ac} \times P_i^{h,-}$ electric energy.

Let us now express T_i^r as a function of the time, P_i^h and T_i^{out}.

With (15), (16), (17), (18), (19) :

$$\forall\, t \in d_i, \quad \frac{dT_i^r(t)}{dt} = \frac{1}{M_{air}.c}\left\{P_i^h - \frac{T_i^r(t) - T_i^{out}}{R_{eq}}\right\}$$

Then :

$$\forall\, t \in d_i, \quad \frac{dT_i^r(t)}{dt} + \frac{T_i^r(t)}{M_{air}.c.R_{eq}} = \frac{P_i^h}{M_{air}.c} + \frac{T_i^{out}}{M_{air}.c.R_{eq}} \tag{20}$$

Solving the differential equation for T_i^r with the origin of time being taken at the beginning of d_i :

$$\forall\, t \in d_i,\quad T_i^r(t) = \left(T_i^{r,0} - R_{eq}P_i^h - T_i^{out}\right)\exp\left(\frac{-t}{M_{air}.c.R_{eq}}\right) + R_{eq}P_i^h + T_i^{out} \tag{21}$$

T^r is continuous, then T_i^r is continuous and the continuity at the boundary between d_i and d_{i+1} gives :

$$T_i^r(\tau) = T_{i+1}^{r,0} \tag{22}$$

Combining (21) and (22) :

$$\left(T_i^{r,0} - R_{eq}P_i^h - T_i^{out}\right)\exp\left(\frac{-\tau}{M_{air}.c.R_{eq}}\right) + R_{eq}P_i^h + T_i^{out} = T_{i+1}^{r,0}$$

Hence :

$$T_i^{r,0}\exp\left(\frac{-\tau}{M_{air}.c.R_{eq}}\right) - T_{i+1}^{r,0} + R_{eq}\left(1 - \exp\left(\frac{-\tau}{M_{air}.c.R_{eq}}\right)\right)P_i^h = \left(\exp\left(\frac{-\tau}{M_{air}.c.R_{eq}}\right) - 1\right)T_i^{out}$$

Define $K_\tau = \exp\left(\frac{-\tau}{M_{air}.c.R_{eq}}\right)$

Finally :

$$\boxed{\forall\, i \in [\![1, N]\!],\quad T_i^{r,0}K_\tau - T_{i+1}^{r,0} + R_{eq}(1 - K_\tau)P_i^h = (K_\tau - 1)T_i^{out} \qquad (23)}$$

Let now $\boldsymbol{A_{eq}^{(II)}}$, $\boldsymbol{B_{eq}^{(II)}}$ and $\boldsymbol{X^{(II)}}$ be:

$$\boldsymbol{A_{eq}^{(II)}} =$$

$$\begin{array}{cc}
N & \begin{bmatrix}
K_\tau & -1 & 0 & 0 & \dots & 0 & 0 & 0 & R_{eq}(1-K_\tau) & -R_{eq}(1-K_\tau) & 0 & 0 & \dots & \dots & 0 & 0 \\
0 & K_\tau & -1 & 0 & & & \vdots & \vdots & 0 & 0 & R_{eq}(1-K_\tau) & -R_{eq}(1-K_\tau) & & & \vdots & \vdots \\
0 & 0 & \ddots & & \ddots & & \vdots & \vdots & \vdots & \vdots & 0 & 0 & & & \vdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & \ddots & -1 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\
\vdots & \vdots & & & \ddots & K_\tau & -1 & 0 & \vdots & \vdots & \vdots & \vdots & & & 0 & 0 \\
0 & 0 & \dots & \dots & \dots & 0 & K_\tau & -1 & 0 & 0 & 0 & 0 & \dots & & R_{eq}(1-K_\tau) & -R_{eq}(1-K_\tau)
\end{bmatrix} \\
& \qquad N+1 \qquad\qquad\qquad\qquad\qquad\qquad 2N
\end{array}$$

$$\boldsymbol{B_{eq}^{(II)}} = (K_\tau - 1)\begin{bmatrix} T_1^{out} \\ \vdots \\ T_i^{out} \\ \vdots \\ T_N^{out} \end{bmatrix} \qquad \boldsymbol{X^{(II)}} = \begin{bmatrix} T_1^{r,0} \\ \vdots \\ T_{N+1}^{r,0} \\ P_1^{h,+} \\ P_1^{h,-} \\ \vdots \\ P_N^{h,+} \\ P_N^{h,-} \end{bmatrix}$$

Then:

$$\boxed{\text{(23) is equivalent to } \boldsymbol{A_{eq}^{(II)}}\boldsymbol{X^{(II)}} = \boldsymbol{B_{eq}^{(II)}}}$$

(13) and (14) gives :

$$\forall\, i \in [\![1, N]\!], \qquad T^{h,min} \le T_i^h \le T^{h,max} \tag{24}$$

$$\forall\, i \in [\![1, N]\!], \qquad R^{min} \le T_{i+1}^{r,0} - T_i^{r,0} \le R^{max} \tag{25}$$

then, with (19) :

$$\forall\, i \in [\![1, N]\!], \qquad T^{h,min} \le \frac{P_i^h}{\dot{M}.c} + T_i^{r,0} \le T^{h,max}$$

$$\forall\, i \in [\![1, N-1]\!], \qquad -R^{min} \le \left(\frac{P_{i+1}^h}{\dot{M}.c} + T_{i+1}^{r,0}\right) - \left(\frac{P_i^h}{\dot{M}.c} + T_i^{r,0}\right) \le R^{max}$$

And finally :

$$\forall\, i \in [\![1, N]\!], \qquad T_i^{r,0} + \frac{P_i^{h,+}}{\dot{M}.c} - \frac{P_i^{h,-}}{\dot{M}.c} \leq T^{h,max} \tag{26}$$

$$\forall\, i \in [\![1, N]\!], \qquad -T_i^{r,0} - \frac{P_i^{h,+}}{\dot{M}.c} + \frac{P_i^{h,-}}{\dot{M}.c} \leq -T^{h,min} \tag{27}$$

$$\forall\, i \in [\![1, N-1]\!], \qquad -T_i^{r,0} + T_{i+1}^{r,0} - \frac{P_i^{h,+}}{\dot{M}.c} + \frac{P_i^{h,-}}{\dot{M}.c} + \frac{P_{i+1}^{h,+}}{\dot{M}.c} - \frac{P_{i+1}^{h,-}}{\dot{M}.c} \leq R^{max} \tag{28}$$

$$\forall\, i \in [\![1, N-1]\!], \qquad +T_i^{r,0} - T_{i+1}^{r,0} + \frac{P_i^{h,+}}{\dot{M}.c} - \frac{P_i^{h,-}}{\dot{M}.c} - \frac{P_{i+1}^{h,+}}{\dot{M}.c} + \frac{P_{i+1}^{h,-}}{\dot{M}.c} \leq R^{min} \tag{29}$$

Let now $\boldsymbol{A}^{(\boldsymbol{II})}$ and $\boldsymbol{B}^{(\boldsymbol{II})}$ be :

$$\boldsymbol{A}^{(\boldsymbol{II})} = \begin{bmatrix} \boldsymbol{I_N} & \boldsymbol{0}_{\mathbb{R}^N} & \frac{1}{\dot{M}.c}\boldsymbol{F_N} \\ -\boldsymbol{I_N} & \boldsymbol{0}_{\mathbb{R}^N} & \frac{-1}{\dot{M}.c}\boldsymbol{F_N} \\ \boldsymbol{G_{N-1}} & \boldsymbol{0}_{\mathbb{R}^N} & \frac{1}{\dot{M}.c}\boldsymbol{H_{N-1}} \\ -\boldsymbol{G_{N-1}} & \boldsymbol{0}_{\mathbb{R}^N} & \frac{-1}{\dot{M}.c}\boldsymbol{H_{N-1}} \end{bmatrix} \qquad \boldsymbol{B}^{(\boldsymbol{II})} = \begin{bmatrix} T^{h,max}.\,\boldsymbol{1}_{\mathbb{R}^N} \\ -T^{h,min}.\,\boldsymbol{1}_{\mathbb{R}^N} \\ R^{max}.\,\boldsymbol{1}_{\mathbb{R}^{N-1}} \\ R^{min}.\,\boldsymbol{1}_{\mathbb{R}^{N-1}} \end{bmatrix}$$

With :

$$\boldsymbol{F_N} = \begin{array}{c} \overset{2N}{\longleftrightarrow} \\ \begin{bmatrix} 1 & -1 & 0 & 0 & \dots & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & & & & & & \\ & & & & \ddots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & & & & \\ & & & & & & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & \dots & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} \updownarrow N$$

$$\boldsymbol{G_{N-1}} = \begin{array}{c} \overset{N}{\longleftrightarrow} \\ \begin{bmatrix} -1 & 1 & 0 & \dots & \dots & \dots & 0 \\ 0 & -1 & 1 & & & & \vdots \\ \vdots & & \ddots & \ddots & & & \vdots \\ \vdots & & & \ddots & \ddots & & \vdots \\ \vdots & & & & -1 & 1 & 0 \\ 0 & \dots & \dots & \dots & 0 & -1 & 1 \end{bmatrix} \end{array} \updownarrow N-1$$

$$\boldsymbol{H_{N-1}} = \begin{array}{c} \overset{2N}{\longleftrightarrow} \\ \begin{bmatrix} -1 & 1 & 1 & -1 & 0 & 0 & & \dots & \dots & & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & & & & & \vdots & \vdots \\ \vdots & \vdots & & & & & \ddots & & & & \vdots & \vdots \\ \vdots & \vdots & & & & & & \ddots & 1 & -1 & 0 & 0 \\ 0 & 0 & & & \dots & & & \dots & -1 & 1 & 1 & -1 \end{bmatrix} \end{array} \updownarrow N-1$$

Then:

$$\boxed{\text{(26),(27),(28),(29) are equivalent to } \boldsymbol{A^{(II)}X^{(II)}} \leq \boldsymbol{B^{(II)}}}$$

$\boldsymbol{\mathcal{P}^u} = [p_1^u \quad p_2^u \quad \dots \quad p_{N-1}^u \quad p_N^u]^T$ is still defined as the electricity prices corresponding to the subdivision $\mathcal{d}$.

Define $\widetilde{\boldsymbol{\mathcal{P}}}^{(\boldsymbol{II})} = \left[\boldsymbol{0}_{\mathbb{R}^{N+1}}{}^T \quad \eta^h p_1^u \quad \eta^{ac} p_1^u \quad \dots \quad \eta^h p_N^u \quad \eta^{ac} p_N^u\right]^T$

Note that we have assumed that the power consumed by the heating/AC was directly provided by the utility grid.

The interruptible loads optimal schedule problem can finally be expressed as the following linear programming problem:

Minimize	$\mathcal{C}^{(II)} = \widetilde{\boldsymbol{\mathcal{P}}}^{(\boldsymbol{II})^T} . \boldsymbol{X^{(II)}}$	
Subject to	$\boldsymbol{A^{(II)}X^{(II)}} \leq \boldsymbol{B^{(II)}}$	(LP-II)
	$\boldsymbol{A_{eq}^{(II)}X^{(II)}} = \boldsymbol{B_{eq}^{(II)}}$	
	$\forall\, i \in [\![1, N]\!],\ T^{r,min} \leq T_i^{r,0} \leq T^{r,max}$	

Note about the operation of the heating/AC system:

Equation (19) shows that if $T_i^h = T_i^{r,0}$ then $P_i^h = 0$, which means physically that the heating/AC does not exchange heat with the room over d_i.

Then, in situations where we will have $T_i^h - T_i^{r,0} < \varepsilon$ with ε a "small" quantity to be defined when implementing the algorithm, we will conclude that the heating/AC is turned off over d_i.

5.3 - Simulations

5.3.1- Introduction

To conclude this chapter, we simulate successively a winter scenario and a summer scenario in order to illustrate the mathematical theory discussed above. For each scenario, the system (LP-II) is solved for the home grid defined in Table 3.

TABLE 3
HOMEGRID PARAMETERS – CHAPTER 5

Symbol	Quantity	Assumed value
D	Time horizon	24h
N	Number of time intervals	288
τ	Length of each time interval	5min
η^h	Heating efficiency	1
η^{ac}	AC efficiency	1
$\dot{M}$	Air flow rate	1 kg/s
$T^{h,min}$	Minimum heater-AC temperature	5°C
$T^{h,max}$	Maximum heater-AC temperature	32°C
R^{min}	Slew rate, temperature decrease	6°C
R^{max}	Slew rate, temperature increase	6°C
T^{init}	Initial room temperature	20°C
ε	ON/OFF status threshold	1°C

Recall that the storage system and the generation capabilities are off-line in this chapter. The utility grid is then the only source providing power to the heating/AC.

5.3.2- Winter scenario

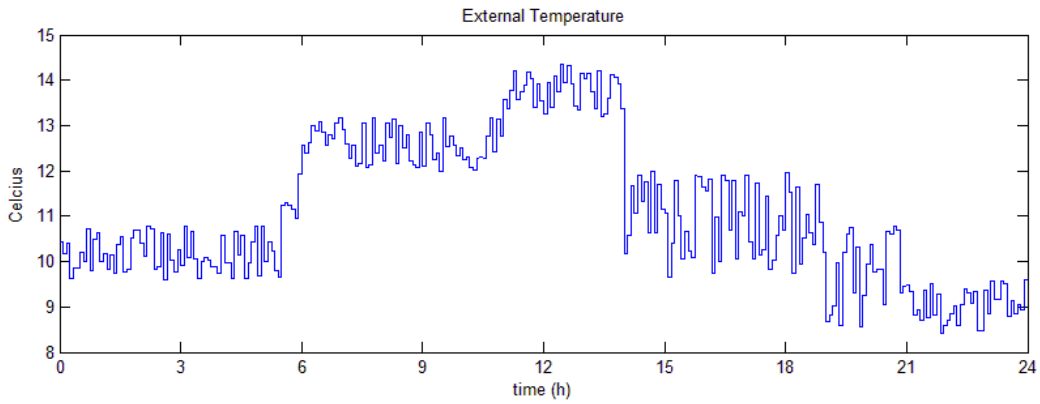


FIGURE 20: External temperature forecasts– winter scenario

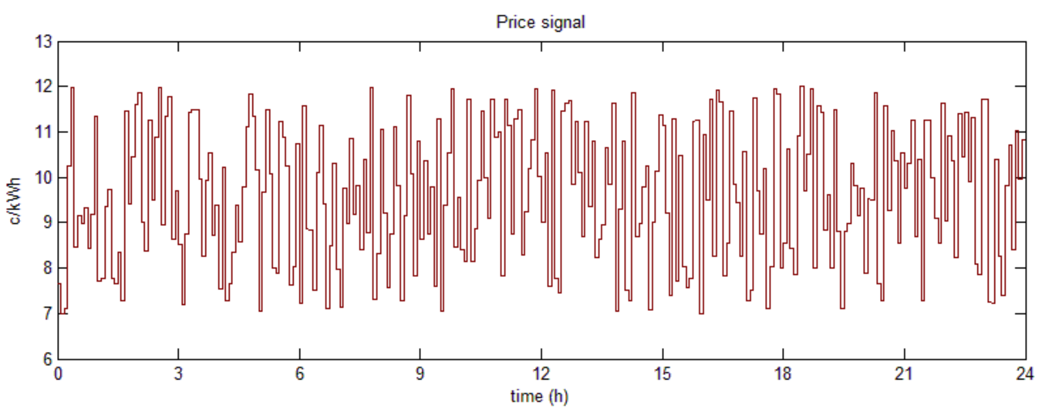


FIGURE 21: Price forecasts – winter scenario

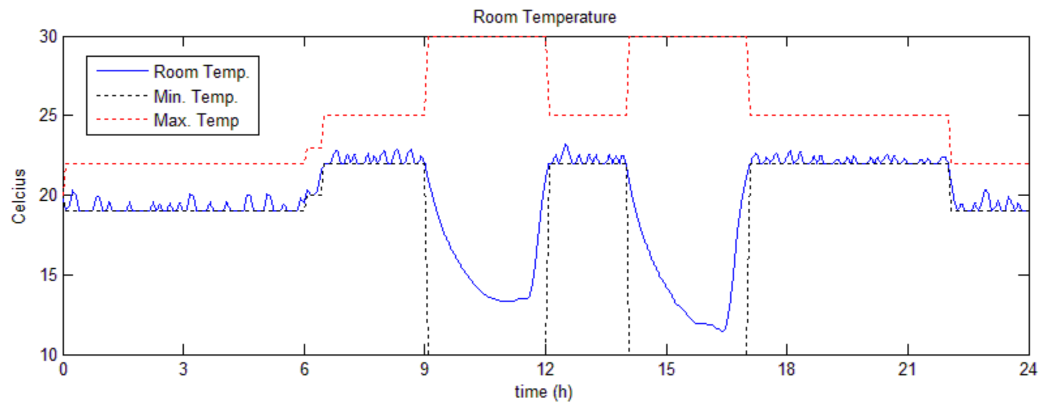


FIGURE 22: Room temperature – winter scenario

Notice how the controller lets the room temperature decreasing when the user's comfort preferences are relaxed in order to save energy (for instance from 9am to 12am). The forecasted cost for the next day is $1.1.

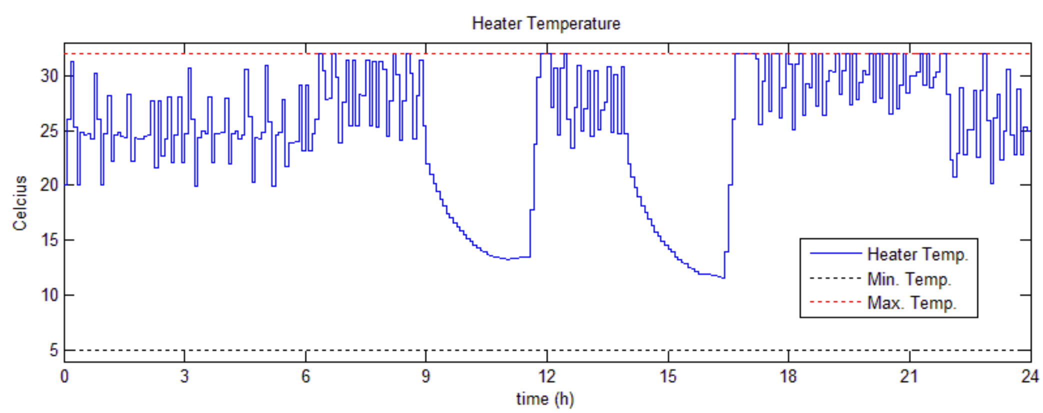


FIGURE 23: Heater temperature – winter scenario

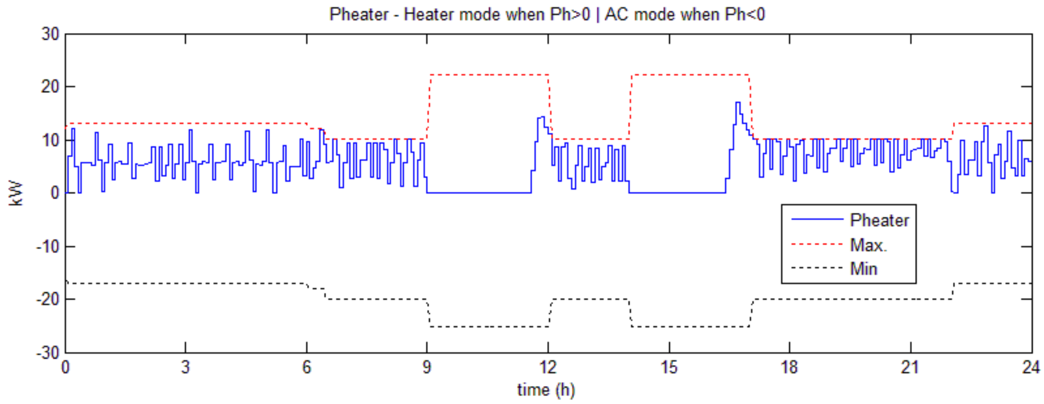


FIGURE 24: Heater power consumption – winter scenario

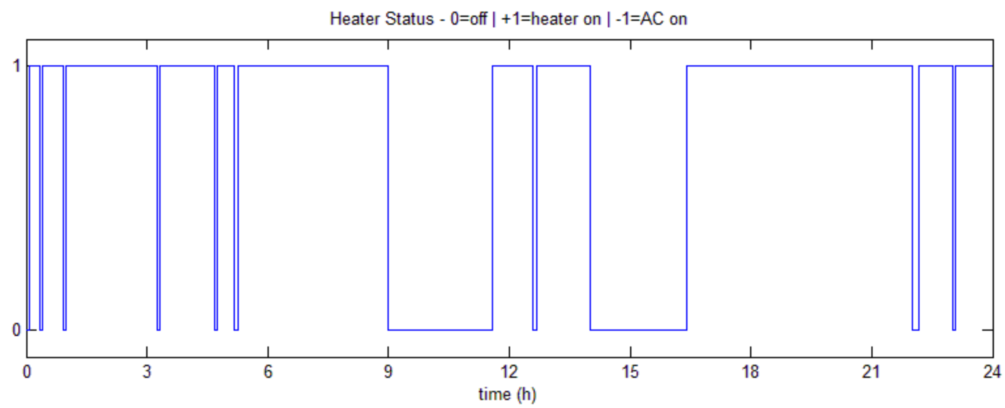


FIGURE 25: Heater status – winter scenario

5.3.3- Summer scenario

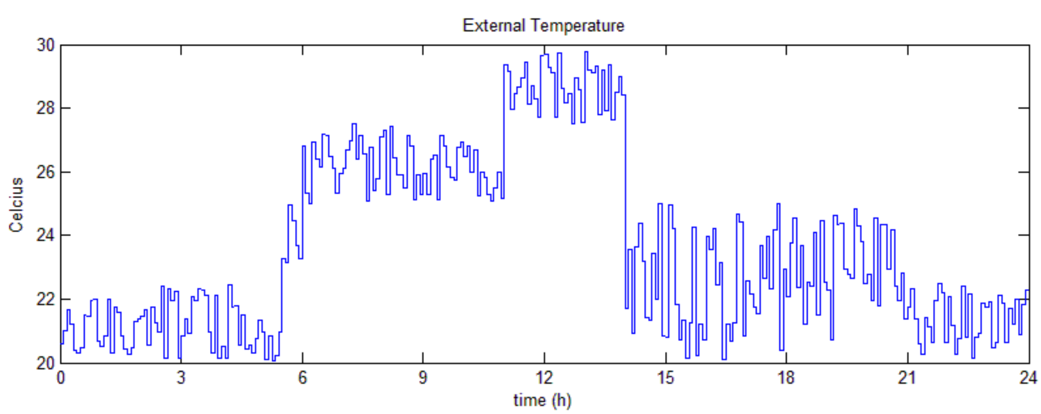


FIGURE 26: External temperature forecasts – summer scenario

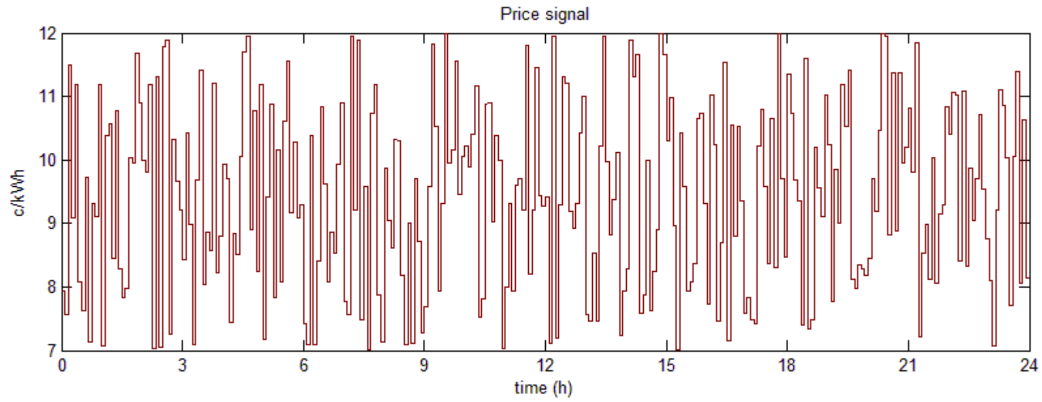


FIGURE 27: Price forecasts – summer scenario

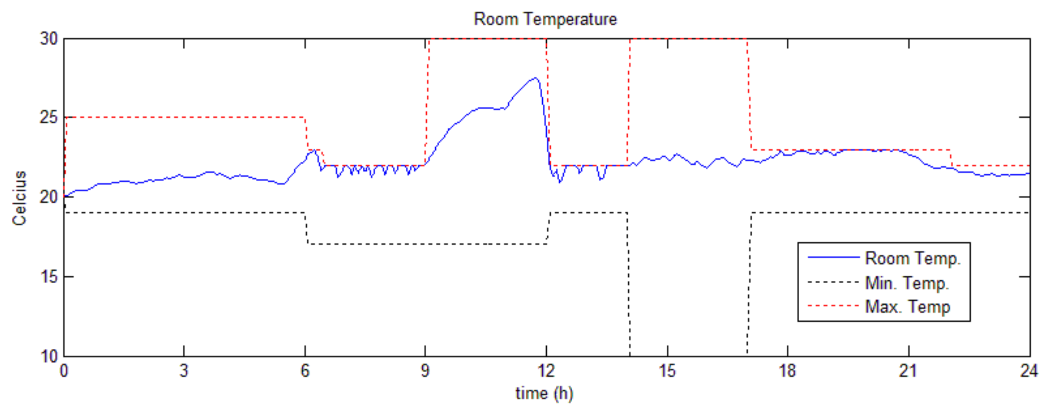


FIGURE 28: Room temperature – summer scenario

Similarly to what happened in the winter scenario, the controller lets the room temperature increasing when the user's comfort preferences are relaxed (for instance from 9am to 12am). Notice that from 2pm to 5pm, because the electricity price is lower on average than around 6pm, the room temperature is maintained close to 24°C.

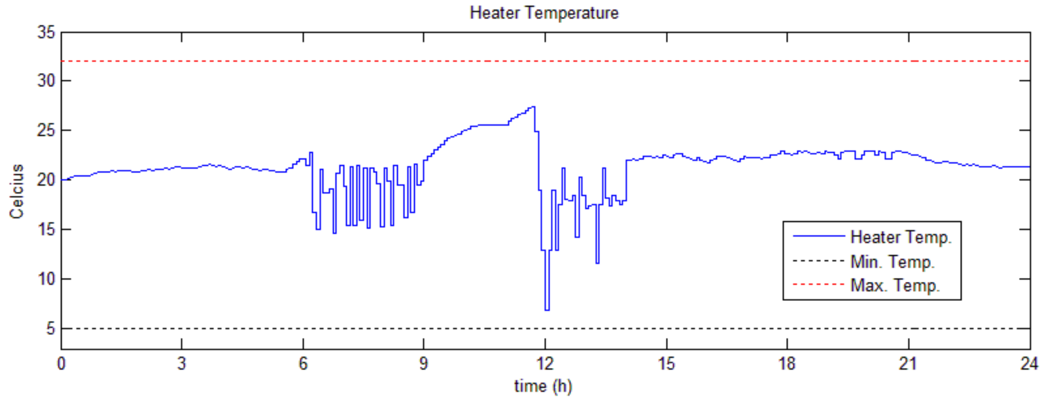


FIGURE 29: Heater temperature – summer scenario

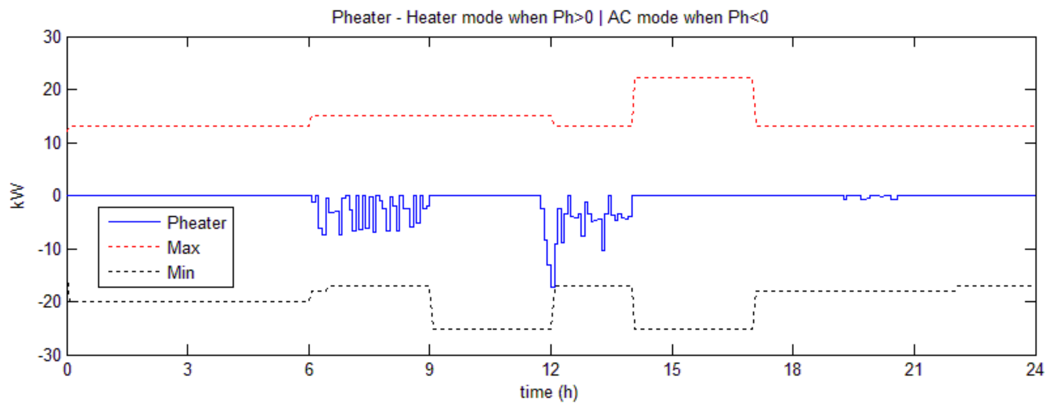


FIGURE 30: Heater power consumption – summer scenario

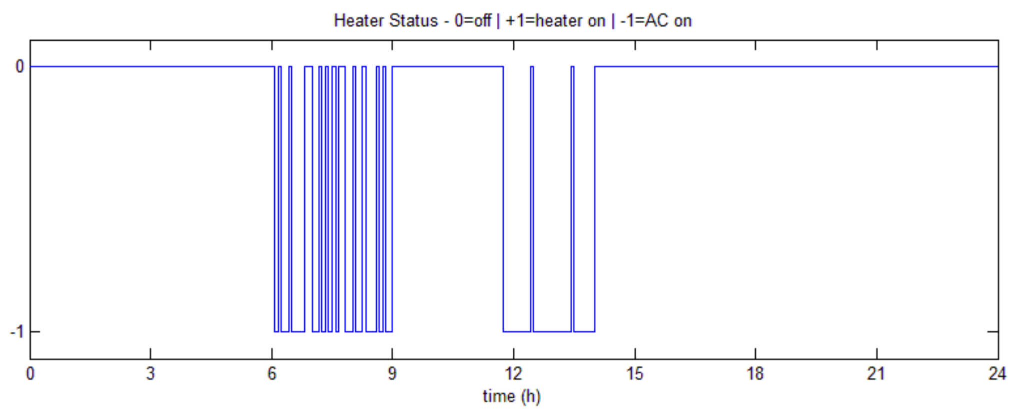


FIGURE 31: Heater status – summer scenario

For the summer scenario, the forecasted cost for the next day is $1.86.

CHAPTER 6

THE GENERAL CASE

6.1 - Introduction

In this chapter, we now combine the two controllable loads groups, the non-interruptible loads and the interruptible loads, into one single system, similarly to what we did in (LPI) for the non-interruptible loads, and (LPII) for the interruptible loads.

The storage system and the distributed generation units are now connected to the home grid again.

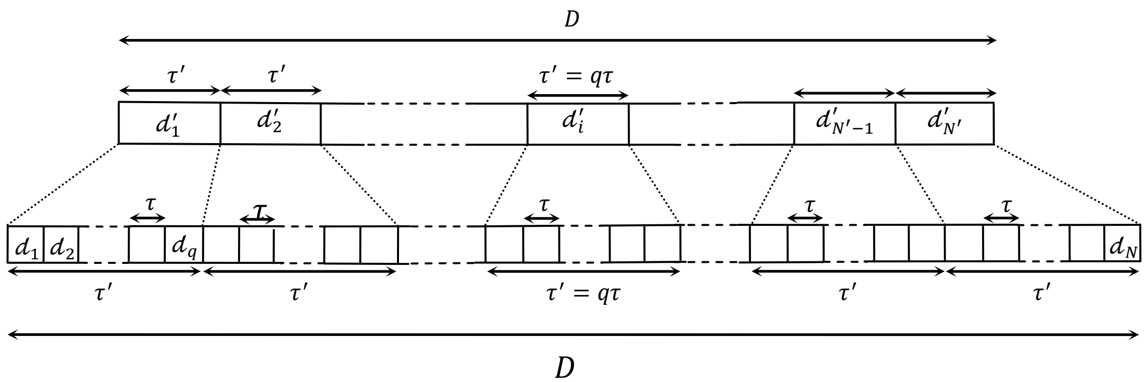
6.2 - Mathematical formulation as a linear programming problem

First, notice that a subdivision adapted to the non-interruptible loads (washing machine, dryer, dish washer, etc), is not necessarily adapted to the interruptible loads (heating, A/C).

For instance, with $D = 24\,h$, choosing $N = 48$ allows to divide D into intervals of length $\tau = 30\,\mathrm{min}$. This means approaching the cycles of the non-interruptible loads by intervals of the type $n\tau, n \in \mathbb{N}$ ie 30min, 60min, 90min, etc. This subdivision seems adapted to the non-interruptible loads, but may not be adapted to the heating and A/C. Indeed one of the assumptions we made is that $\tau \ll \tau_{T^{out}}$ and $\tau \ll \tau_{T_i^r}$, but with $\tau = 30\,\mathrm{min}$, T^{out} may vary significantly over that period of time, leading to significant variations on T_i^r as well.

Then, we see the impossibility to combine directly (LP-I) and (LP-II), as we need first to find a subdivision which is adapted to both the interruptible and non-interruptible loads.

A possible solution is to divide each interval of the initial subdivision into a new subdivision of q intervals of equal length: the initial subdivision remains adapted to the non-interruptible loads, and we choose the new subdivision such that it is consistent to the interruptible loads.



We have the following relations :

$\tau' = q\tau$

$N'q = N$

The variables in $\boldsymbol{X}^{(\boldsymbol{I})}$ and $\boldsymbol{X}^{(\boldsymbol{II})}$ remain the same, except that each $P_i^{(g)}$ in $\boldsymbol{X}^{(\boldsymbol{I})}$ is replaced by q variables $P_{i,1}^{(g)}, \dots, P_{i,q}^{(g)}$ to fit the new subdivision d. All the $P_{i,j}^{(g)}$ are then re-indexed as $P_1^{(g)}, \dots, P_N^{(g)}$.

Define:

$$\boldsymbol{X} = \begin{bmatrix} P_1^{(g)} & \dots & P_N^{(g)} & \delta_1^{bt} & \dots & \delta_N^{bt} & \delta_1^{L_1} & \dots & \delta_{S_{L_1}}^{L_1} & \dots & \dots & \delta_1^{L_\ell} & \dots & \delta_{S_{L_\ell}}^{L_\ell} & T_1^{r,0} & \dots & T_{N+1}^{r,0} & P_1^{h,+} & P_1^{h,-} & \dots & P_N^{h,+} & P_N^{h,-} \end{bmatrix}^T$$

$$
\boldsymbol{A} = \begin{bmatrix}
-\tau \boldsymbol{T_N} & -\tau P_{nom}^{bt} \boldsymbol{T_N} & \tau P_1^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_1}} & \tau P_2^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_2}} & \dots & \tau P_\ell^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_\ell}} & \boldsymbol{0}_{\mathbb{R}^{N\times N}} & \boldsymbol{0}_{\mathbb{R}^{N}} & \tau \boldsymbol{\eta^\alpha} \\
\tau \boldsymbol{T_N} & \tau P_{nom}^{bt} \boldsymbol{T_N} & -\tau P_1^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_1}} & -\tau P_2^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_2}} & \dots & -\tau P_\ell^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_\ell}} & \boldsymbol{0}_{\mathbb{R}^{N\times N}} & \boldsymbol{0}_{\mathbb{R}^{N}} & -\tau \boldsymbol{\eta^\alpha} \\
\tau \boldsymbol{I_N} & \tau P_{nom}^{bt} \boldsymbol{I_N} & -\tau P_1^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_1}} & -\tau P_2^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_2}} & \dots & -\tau P_\ell^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_\ell}} & \boldsymbol{0}_{\mathbb{R}^{N\times N}} & \boldsymbol{0}_{\mathbb{R}^{N}} & -\tau \boldsymbol{\eta^\beta} \\
-\tau \boldsymbol{I_N} & -\tau P_{nom}^{bt} \boldsymbol{I_N} & \tau P_1^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_1}} & \tau P_2^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_2}} & \dots & \tau P_\ell^L . \boldsymbol{M}_{\boldsymbol{N',q}}^{\boldsymbol{\alpha,L_\ell}} & \boldsymbol{0}_{\mathbb{R}^{N\times N}} & \boldsymbol{0}_{\mathbb{R}^{N}} & \tau \boldsymbol{\eta^\beta} \\
 & & \boldsymbol{M_{S_{L_1}}} & & & & & & \\
 & & & \boldsymbol{M_{S_{L_2}}} & & & & & \\
 & & & & \ddots & & & & \\
 & & & & & \boldsymbol{M_{S_{L_\ell}}} & & & \\
 & & & & & & \boldsymbol{I_N} & \boldsymbol{0}_{\mathbb{R}^{N}} & \frac{1}{\dot{M}c} \boldsymbol{F_N} \\
 & & & & & & -\boldsymbol{I_N} & \boldsymbol{0}_{\mathbb{R}^{N}} & \frac{-1}{\dot{M}c} \boldsymbol{F_N} \\
 & & & & & & \boldsymbol{G_{N-1}} & \boldsymbol{0}_{\mathbb{R}^{N-1}} & \frac{1}{\dot{M}c} \boldsymbol{H_{N-1}} \\
 & & & & & & -\boldsymbol{G_{N-1}} & \boldsymbol{0}_{\mathbb{R}^{N-1}} & \frac{-1}{\dot{M}c} \boldsymbol{H_{N-1}}
\end{bmatrix}
$$

$$
\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}^{(\boldsymbol{I})} \\ \boldsymbol{B}^{(\boldsymbol{II})} \end{bmatrix}
$$

With :

$$
\boldsymbol{\eta^\alpha} = \begin{array}{cc}
\overset{2N}{\longleftrightarrow} & \\
\begin{bmatrix}
\eta^h & \eta^{ac} & 0 & 0 & \dots & \dots & \dots & 0 & 0 \\
\eta^h & \eta^{ac} & \eta^h & \eta^{ac} & & & & \vdots & \vdots \\
\vdots & \vdots & & & & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & & & & & 0 & 0 \\
\eta^h & \eta^{ac} & & \dots & & \dots & & \eta^h & \eta^{ac}
\end{bmatrix} & \updownarrow \; N
\end{array}
$$

$$
\boldsymbol{\eta^\beta} = \begin{array}{cc}
\overset{2N}{\longleftrightarrow} & \\
\begin{bmatrix}
\eta^h & \eta^{ac} & 0 & 0 & \dots & \dots & \dots & 0 & 0 \\
0 & 0 & \eta^h & \eta^{ac} & & & & \vdots & \vdots \\
\vdots & \vdots & & & & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & & & & & 0 & 0 \\
0 & 0 & & \dots & & \dots & & \eta^h & \eta^{ac}
\end{bmatrix} & \updownarrow \; N
\end{array}
$$

$$
\boldsymbol{A_{eq}} = \begin{bmatrix}
\boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{I})} & \boldsymbol{0}_{\mathbb{R}^{\ell\times(3N+1)}} \\
\boldsymbol{0}_{\mathbb{R}^{4N\times(2N+\sum_{i=1}^{\ell} S_{L_i})}} & \boldsymbol{A}_{\boldsymbol{eq}}^{(\boldsymbol{II})}
\end{bmatrix}
\qquad\qquad
\boldsymbol{B_{eq}} = \begin{bmatrix} \boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{I})} \\ \boldsymbol{B}_{\boldsymbol{eq}}^{(\boldsymbol{II})} \end{bmatrix}
$$

Define $\boldsymbol{\mathcal{P}^u} = [p_1^u \quad p_2^u \quad \dots \quad p_{N-1}^u \quad p_N^u]^T$ the electricity prices corresponding to the subdivision d.

Define $\boldsymbol{\mathcal{P}^f} = p^f \mathbf{1}_{\mathbb{R}^N}{}^T$ as the biofuel prices corresponding to the subdivision d. Biofuel prices are supposed constant over D and equal to p^f.

Finally, define $\widetilde{\boldsymbol{\mathcal{P}}} = \big[\boldsymbol{\mathcal{P}^u}^T \quad P_{nom}^{bt}\boldsymbol{\mathcal{P}^f}^T \quad \underbrace{0 \quad \dots \quad 0}_{\left(\sum_{i=1}^{\ell} S_{L_i}\right)+3\text{N}+1}\big]^T$

Note that P_{nom}^{bt} has to be included in $\widetilde{\boldsymbol{\mathcal{P}}}$ since $\boldsymbol{X}$ only contains the logic numbers $\delta_1^{bt}, \dots, \delta_N^{bt}$ defining if the biofuel-burning turbine is on or off for each time interval d_i.

The optimization problem combining the non-interruptible loads and the interruptible loads can finally be expressed as the following mixed-integer linear programming problem:

Minimize	$\mathcal{C} = \widetilde{\boldsymbol{\mathcal{P}}}^T.\boldsymbol{X}$	
Subject to		
	$\boldsymbol{AX} \leq \boldsymbol{B}$	
	$\boldsymbol{A_{eq}X} = \boldsymbol{B_{eq}}$	(LP)
	$\forall i \in [\![1, N]\!],\ \delta_i^{bt} \in [\![0,1]\!]$	
	$\forall i \in [\![1, \ell]\!], \forall j \in [\![1, S_{L_i}]\!],\ \delta_j^{L_i} \in [\![0,1]\!]$	
	$\forall i \in [\![1, N]\!], T^{r,min} \leq T_i^{r,0} \leq T^{r,max}$	

6.3 – Simulations

6.3.1- Introduction

To conclude this chapter, we simulate a scenario involving both interruptible and non-interruptible loads in order to illustrate the mathematical theory discussed above. The system (LP) is solved for the home grid defined in Table 4.

6.3.2- Scenario simulation

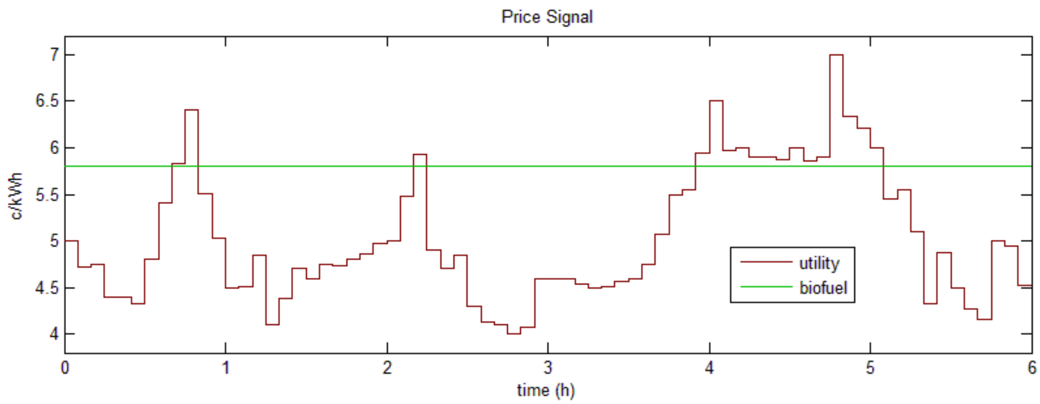


FIGURE 32: Price forecasts –general case

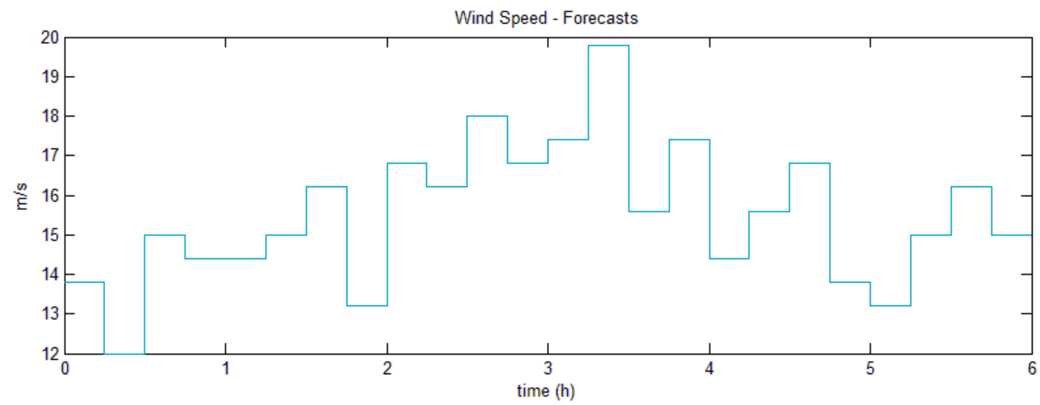


FIGURE 33: Wind speed forecasts – general case

TABLE 4
HOMEGRID PARAMETERS – CHAPTER 6

Symbol	Quantity	Assumed value
D	Time horizon	6h
N'	Number of time intervals – subdivision d'	18
τ'	Length of each time interval – subdivision d	20min
q	Subdivision factor	4
N	Number of time intervals	72
τ	Length of each time interval	5min
p^f	Biofuel price	5.8 ¢/kWh
E^{init}	Energy stored at $t = 0$	0 kWh
E^{max}	Maximum capacity - storage device	3 kWh
R^c	Maximal charging rate - storage device	0.5 kWh
R^d	Maximal discharging rate - storage device	0.5 kWh
P_{nom}^{bt}	Nominal power - biofuel-burning turbine	1.5 kW
$\eta^{(s)}$	Solar panel efficiency	0.15
$S^{(s)}$	Solar panel eq. surface	1m^2
$\eta^{(w)}$	Wind turbine efficiency	0.20
$S^{(w)}$	Wind turbine eq. surface	1m^2
ℓ	Number of non-interruptible loads	6
P_1^L	Washing machine consumption	1 kW
P_2^L	Dryer consumption	1.5 kW
P_3^L	Dishwasher consumption	1 kW
P_4^L	Defrost cycle consumption	1.3 kW
P_5^L	Water heater consumption	2 kW
P_6^L	PHEV battery consumption	3 kW
k_{L_1}	$k_{L_1}\tau$ is the cycle of L_1	2
k_{L_2}	$k_{L_2}\tau$ is the cycle of L_2	3
k_{L_3}	$k_{L_3}\tau$ is the cycle of L_3	2
k_{L_4}	$k_{L_4}\tau$ is the cycle of L_4	1
k_{L_5}	$k_{L_5}\tau$ is the cycle of L_5	1
k_{L_6}	$k_{L_6}\tau$ is the cycle of L_6	1
γ^{L_1}	L_1 to be scheduled γ^{L_1} times	1
γ^{L_2}	L_2 to be scheduled γ^{L_2} times	1
γ^{L_3}	L_3 to be scheduled γ^{L_3} times	1
γ^{L_4}	L_4 to be scheduled γ^{L_4} times	2
γ^{L_5}	L_5 to be scheduled γ^{L_5} times	2
γ^{L_6}	L_6 to be scheduled γ^{L_6} times	4
η^h	Heating efficiency	1
η^{ac}	AC efficiency	1
$\dot{M}$	Air flow rate	1 kg/s
$T^{h,min}$	Minimum heater-AC temperature	5°C
$T^{h,max}$	Maximum heater-AC temperature	32°C
R^{min}	Slew rate, temperature decrease	5°C
R^{max}	Slew rate, temperature increase	5°C
T^{init}	Initial room temperature	22°C
ε	ON/OFF status threshold	1°C

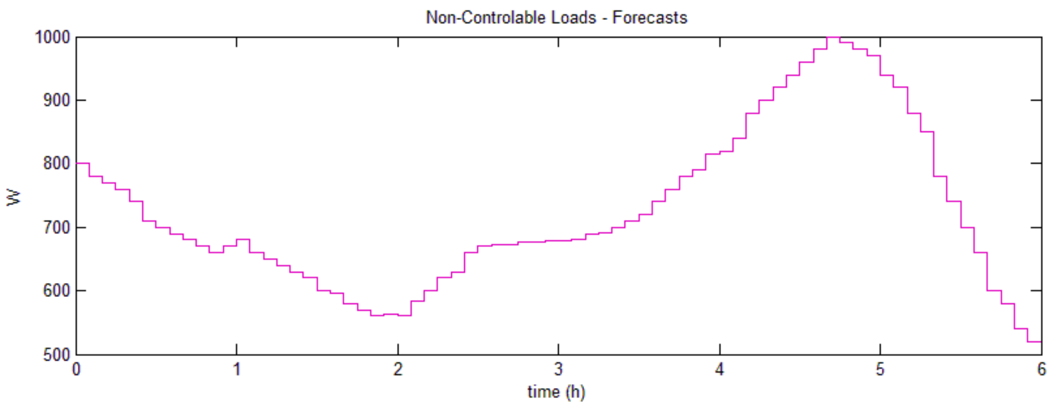


FIGURE 34: Non-controllable loads forecasts – general case

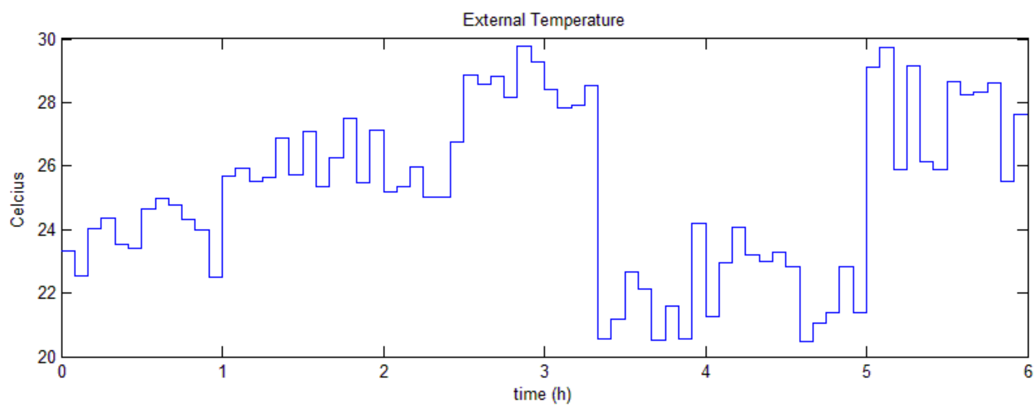


FIGURE 35: External temperature forecasts – general case

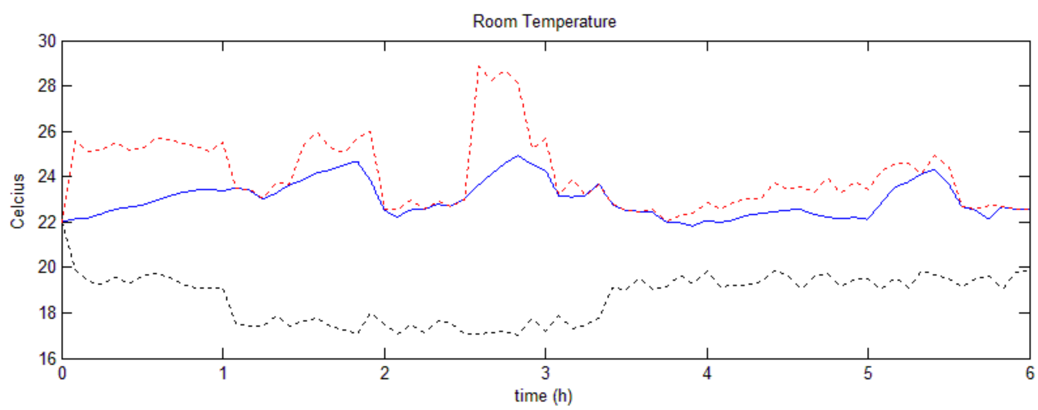


FIGURE 36: Room temperature – general case

Similarly to the simulation results for the summer scenario discussed in chapter 5, notice how the controller lets the room temperature increasing when the user's comfort preferences are relaxed (for instance from hour 2.5 to hour 3). Notice than from hour 0 to hour 1, because the price is relatively low compared to the next period, the controller keeps the room temperature fairly low compared to the maximum limit allowed by the user for that time period.

From hour 4 to hour 5, the room temperature is maintained fairly constant in prevision of the sudden increase in external temperature occurring at hour 5.

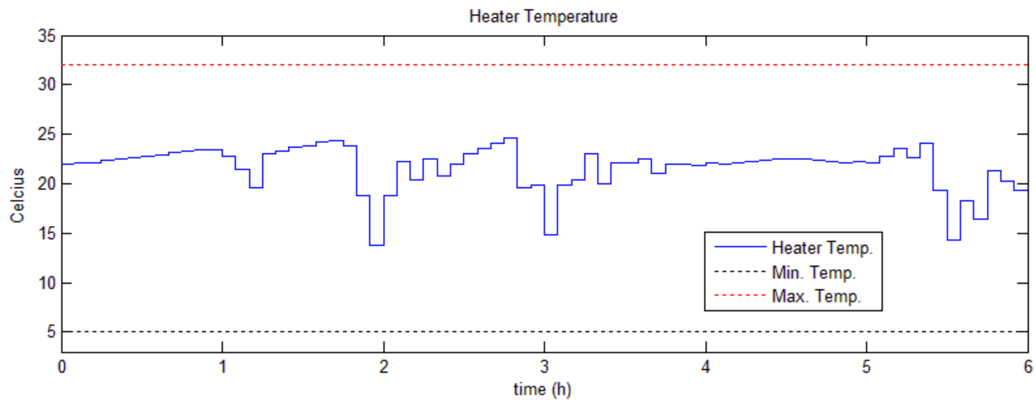


FIGURE 37: Heater temperature – general case

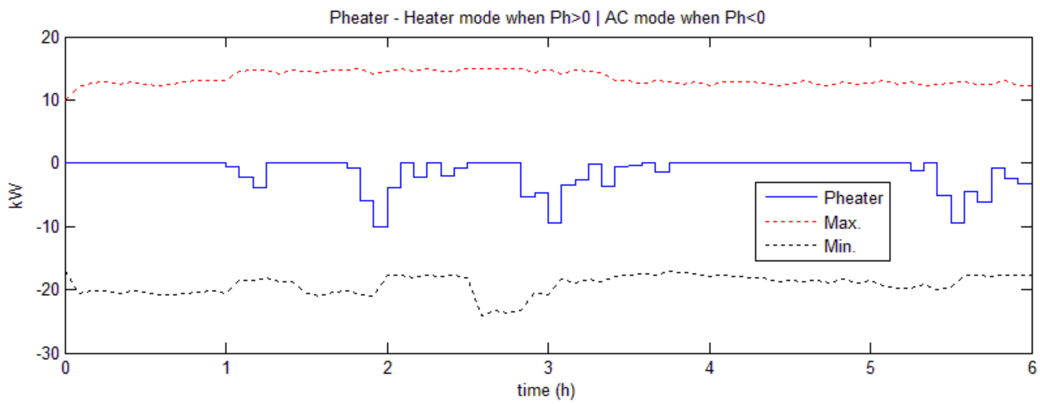


FIGURE 38: Heater power consumption – general case

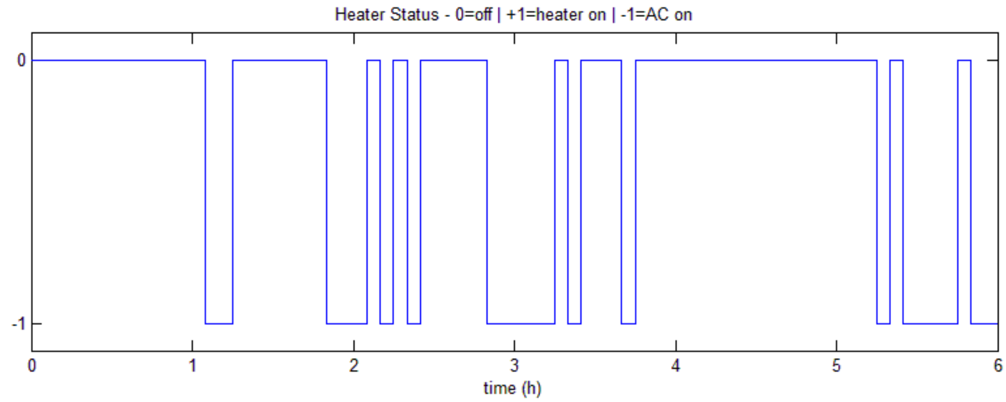


FIGURE 39: Heater status – general case

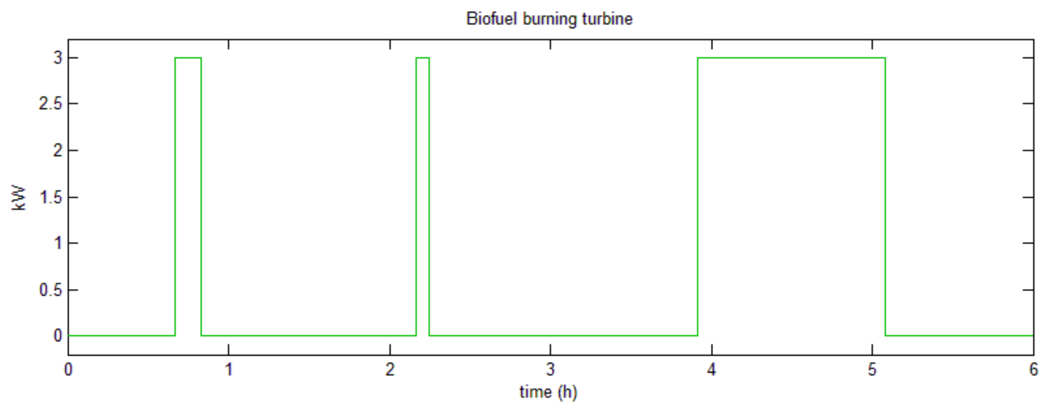


FIGURE 40: Biofuel-burning turbine schedule – general case

Similarly to what we observed in chapter 4, the biofuel-burning turbine starts as soon as the utility price signal goes above the biofuel price.

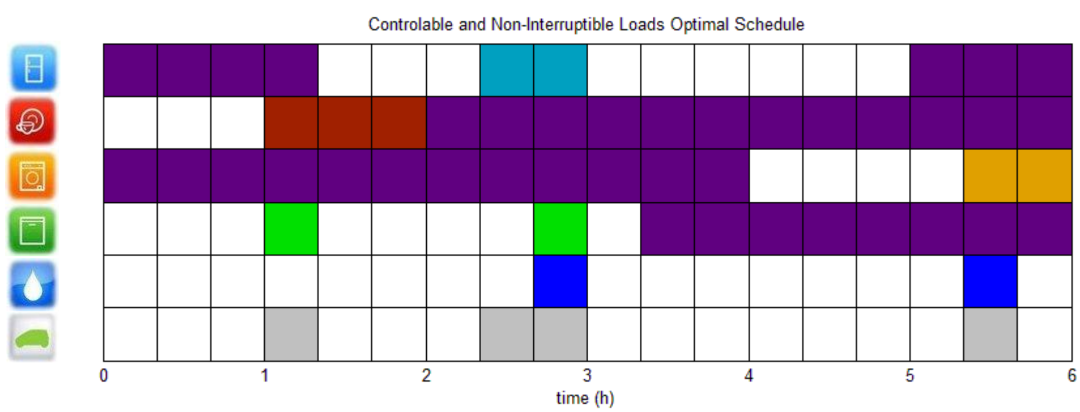


FIGURE 41: Optimal loads schedule – general case

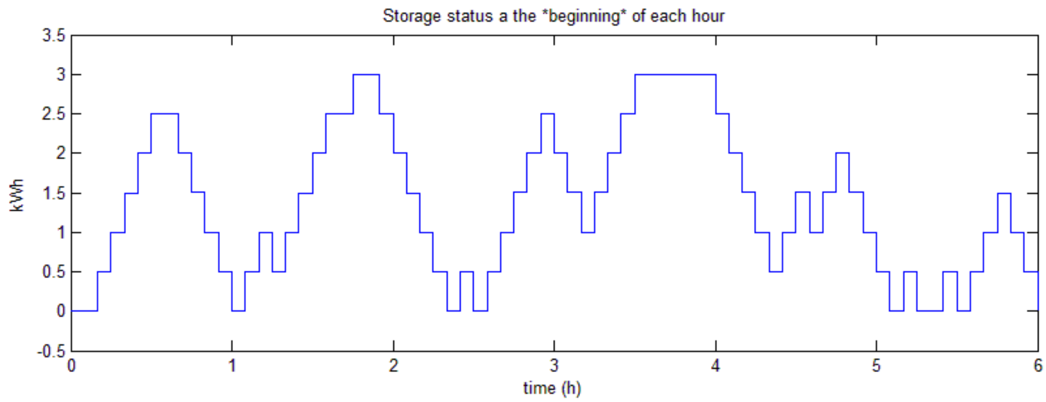


FIGURE 42: Storage schedule – general case

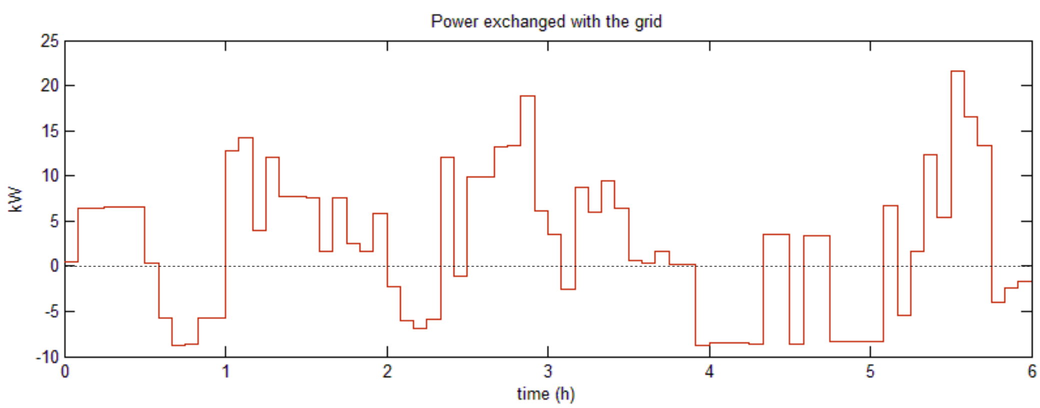


FIGURE 43: Power exchanged with the grid – general case

CHAPTER 7

EXTENSION TO THE CASE OF SEVERAL ENERGY PROVIDERS

7.1 - Introduction

In this chapter, we assume that $\mathcal{u}$ energy providers $U_1, U_2, \ldots, U_{\mathcal{u}}$ are now serving the residential customer. Then, for each d_i, the residential customer has now the ability to deal with one of these providers based on their price signals.

The software now not only has to find the optimal load schedule, but has also to decide which utilities to deal with and when.

7.2 - Mathematical formulation as a linear programming problem

The N variables $P_1^{(g)}, \ldots, P_N^{(g)}$ are now replaced by $\mathcal{u} \times N$ variables $P_1^{(g,1)}, \ldots, P_N^{(g,1)}, \ldots, P_1^{(g,\mathcal{u})}, \ldots, P_N^{(g,\mathcal{u})}$.

Define:

$\forall\, i \in [\![1, \mathcal{u}]\!],\ \ \forall\, j \in [\![1, N]\!],\ \ P_j^{(g,i),+} > 0$ and $P_j^{(g,i),-} > 0$ such that $P_j^{(g,i)} = P_j^{(g,i),+} - P_j^{(g,i),-}$.

Let us call $\boldsymbol{X^{\mathcal{u}}}$ the updated vector $\boldsymbol{X}$ in the case of $\mathcal{u}$ energy providers:

$$\boldsymbol{X^{\mathcal{u}}} = \begin{bmatrix} P_1^{(g,1)} & \ldots & P_N^{(g,1)} & \ldots & P_1^{(g,\mathcal{u})} & \ldots & P_N^{(g,\mathcal{u})} & \delta_1^{bt} & \ldots & \delta_N^{bt} & \delta_1^{L_1} & \ldots & \delta_{S_{L_1}}^{L_1} & \ldots & \delta_1^{L_\ell} & \ldots & \delta_{S_{L_\ell}}^{L_\ell} & T_1^{r,0} & \ldots & T_{N+1}^{r,0} & P_1^{h,+} & P_1^{h,-} & \ldots & P_N^{h,+} & P_N^{h,-} \end{bmatrix}^T$$

Recall that by definition :

$$\boldsymbol{F_N} = \overbrace{\begin{bmatrix} 1 & -1 & 0 & 0 & \ldots & \ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & & & & & & \\ & & & & \ddots & & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \ddots & & & & \\ & & & & & & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots & \ldots & 0 & 0 & 1 & -1 \end{bmatrix}}^{2N} \Big\updownarrow N$$

Define $\boldsymbol{K_N}$:

$$
\begin{array}{ll}
\overset{2N}{\longleftrightarrow} & \\
\boldsymbol{K_N} = \left[\begin{array}{cccccccccc}
1 & -1 & 0 & 0 & \dots & \dots & 0 & 0 & 0 & 0 \\
1 & -1 & 1 & -1 & & & & & & \\
 & & & & \ddots & & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & & \ddots & & & & \\
 & & & & & & 1 & -1 & 0 & 0 \\
1 & -1 & 1 & -1 & \dots & \dots & 1 & -1 & 1 & -1
\end{array}\right] & \updownarrow N
\end{array}
$$

Let now $\boldsymbol{A^u}, \boldsymbol{B^u}, \boldsymbol{A^u_{eq}}, \boldsymbol{B^u_{eq}}$ be:

$$
\boldsymbol{A^u} =
$$

$$
\begin{array}{l}
\quad \overset{u \times N}{\longleftrightarrow} \\
\left[\begin{array}{ccccccccccc}
-\tau\boldsymbol{K_N} & \dots & -\tau\boldsymbol{K_N} & -\tau P^{bt}_{nom}\boldsymbol{T_N} & \tau P^L_1.\boldsymbol{M}^{\alpha,L_1}_{N',q} & \tau P^L_2.\boldsymbol{M}^{\alpha,L_2}_{N',q} & \dots & \tau P^L_\ell.\boldsymbol{M}^{\alpha,L_\ell}_{N',q} & \mathbf{0}_{\mathbb{R}^{((N'-k_L)q+k_L)\times N}} & \mathbf{0}_{\mathbb{R}^N} & \tau\boldsymbol{\eta}^{\alpha} \\
\tau\boldsymbol{K_N} & \dots & \tau\boldsymbol{K_N} & \tau P^{bt}_{nom}\boldsymbol{T_N} & -\tau P^L_1.\boldsymbol{M}^{\alpha,L_1}_{N',q} & -\tau P^L_2.\boldsymbol{M}^{\alpha,L_2}_{N',q} & \dots & -\tau P^L_\ell.\boldsymbol{M}^{\alpha,L_\ell}_{N',q} & \mathbf{0}_{\mathbb{R}^{((N'-k_L)q+k_L)\times N}} & \mathbf{0}_{\mathbb{R}^N} & -\tau\boldsymbol{\eta}^{\alpha} \\
\tau\boldsymbol{F_N} & \dots & \tau\boldsymbol{F_N} & \tau P^{bt}_{nom}\boldsymbol{I_N} & -\tau P^L_1.\boldsymbol{M}^{\alpha,L_1}_{N',q} & -\tau P^L_2.\boldsymbol{M}^{\alpha,L_2}_{N',q} & \dots & -\tau P^L_\ell.\boldsymbol{M}^{\alpha,L_\ell}_{N',q} & \mathbf{0}_{\mathbb{R}^{N\times N}} & \mathbf{0}_{\mathbb{R}^N} & -\tau\boldsymbol{\eta}^{\beta} \\
-\tau\boldsymbol{F_N} & \dots & -\tau\boldsymbol{F_N} & -\tau P^{bt}_{nom}\boldsymbol{I_N} & \tau P^L_1.\boldsymbol{M}^{\alpha,L_1}_{N',q} & \tau P^L_2.\boldsymbol{M}^{\alpha,L_2}_{N',q} & \dots & \tau P^L_\ell.\boldsymbol{M}^{\alpha,L_\ell}_{N',q} & \mathbf{0}_{\mathbb{R}^{N\times N}} & \mathbf{0}_{\mathbb{R}^N} & \tau\boldsymbol{\eta}^{\beta} \\
 & & & & \boldsymbol{M}_{S_{L_1}} & & & & & & \\
 & & & & & \boldsymbol{M}_{S_{L_2}} & & & & & \\
 & & & & & & \ddots & & & & \\
 & & & & & & & \boldsymbol{M}_{S_{L_\ell}} & & & \\
 & & & & & & & & \boldsymbol{I_N} & \mathbf{0}_{\mathbb{R}^N} & \frac{1}{\dot{M}c}\boldsymbol{F_N} \\
 & & & & & & & & -\boldsymbol{I_N} & \mathbf{0}_{\mathbb{R}^N} & \frac{-1}{\dot{M}c}\boldsymbol{F_N} \\
 & & & & & & & & \boldsymbol{G_{N-1}} & \mathbf{0}_{\mathbb{R}^{N-1}} & \frac{1}{\dot{M}c}\boldsymbol{H_{N-1}} \\
 & & & & & & & & -\boldsymbol{G_{N-1}} & \mathbf{0}_{\mathbb{R}^{N-1}} & \frac{-1}{\dot{M}c}\boldsymbol{H_{N-1}}
\end{array}\right]
\end{array}
$$

$$
\boldsymbol{B^u} = \boldsymbol{B}
$$

$$
\boldsymbol{A^u_{eq}} = [\mathbf{0}_{\mathbb{R}^{(4N+\ell)\times(u-1)}} \quad \boldsymbol{A_{eq}}] \qquad\qquad \boldsymbol{B^u_{eq}} = \boldsymbol{B_{eq}}
$$

Also define $\forall\, i \in [\![1, \mathcal{u}]\!]$, $\boldsymbol{\mathcal{P}^i} = \begin{bmatrix} p_1^{i,+} & -p_1^{i,-} & \dots & p_N^{i,+} & -p_N^{i,-} \end{bmatrix}^T$

- $p_j^{i,+}$ is the price provider i is willing to sell electricity to the customer over d_j
- $p_j^{i,-}$ is the price provider i is willing to buy electricity from the customer over d_j

We will assume in the following that :

$$\forall\, i \in [\![1, \mathcal{u}]\!], \forall\, j \in [\![1, N]\!], \quad \nexists\, k \in [\![1, \mathcal{u}]\!]\backslash\{i\} \ \ | \ \ p_j^{k,-} > p_j^{i,+}$$

With this condition, we assume that energy providers make contracts at the provider level to exchange power between them, and that their price signals cannot be in such way that residential customers act as third parties which could buy large quantities of power from provider i over d_j to sell this power immediately to provider k which would pay $p_j^{k,-} > p_j^{i,+}$ for it (arbitrage).

Define $\widetilde{\boldsymbol{\mathcal{P}}}^{\boldsymbol{u}} = \Big[\boldsymbol{\mathcal{P}^1}^T \quad \dots \quad \boldsymbol{\mathcal{P}^u}^T \quad P_{nom}^{bt}\boldsymbol{\mathcal{P}^f}^T \quad \underbrace{0 \quad \dots \quad 0}_{\left(\sum_{i=1}^{\ell} S_{L_i}\right)+3\mathrm{N}+1}\Big]^T$

The optimization problem combining the non-interruptible loads and the interruptible loads can finally be expressed as the following mixed-integer linear programming problem:

Minimize *Subject to*	$\mathcal{C} = \widetilde{\boldsymbol{\mathcal{P}}}^{\boldsymbol{u}^T}.\boldsymbol{X^u}$	
	$\boldsymbol{A^u X^u} \leq \boldsymbol{B^u}$ $\boldsymbol{A_{eq}^u X^u} = \boldsymbol{B_{eq}^u}$	(LP-u)
	$\forall i \in [\![1, N]\!],\ \delta_i^{bt} \in [\![0,1]\!]$ $\forall i \in [\![1, \ell]\!], \forall j \in [\![1, S_{L_i}]\!],\ \delta_j^{L_i} \in [\![0,1]\!]$ $\forall\, i \in [\![1, N]\!],\ T^{r,min} \leq T_i^{r,0} \leq T^{r,max}$	

7.3 – Simulations

7.3.1- Introduction

To conclude this chapter, we simulate a scenario involving three energy providers in order to illustrate the mathematical theory discussed above. The system (LP-u) is solved for the home grid defined in Table 5.

Assume the following time constraints:

- Load 1 must run between 1:20 and 5
- Load 2 must run between 0 and 2
- Load 3 must run between 4 and 6
- Load 4 must run between 0 and 3:20

Also assume solar and wind modules are off-line for this simulation.

TABLE 5
HOMEGRID PARAMETERS – CHAPTER 7

Symbol	Quantity	Assumed value
D	Time horizon	6h
N'	Number of time intervals – subdivision d'	18
τ'	Length of each time interval – subdivision d	20min
q	Subdivision factor	4
N	Number of time intervals	72
τ	Length of each time interval	5min
$\mathcal{u}$	Number of energy providers	3
p^f	Biofuel price	5 ¢/kWh
E^{init}	Energy stored at $t = 0$	0 kWh
E^{max}	Maximum capacity - storage device	3 kWh
R^c	Maximal charging rate - storage device	0.5 kWh
R^d	Maximal discharging rate - storage device	0.5 kWh
P_{nom}^{bt}	Nominal power - biofuel-burning turbine	1.5 kW
ℓ	Number of non-interruptible loads	6
P_1^L	Washing machine consumption	1 kW
P_2^L	Dryer consumption	1.5 kW
P_3^L	Dishwasher consumption	1 kW
P_4^L	Defrost cycle consumption	1.3 kW
P_5^L	Water heater consumption	2 kW
P_6^L	PHEV battery consumption	3 kW
k_{L_1}	$k_{L_1}\tau$ is the cycle of L_1	2
k_{L_2}	$k_{L_2}\tau$ is the cycle of L_2	3
k_{L_3}	$k_{L_3}\tau$ is the cycle of L_3	2
k_{L_4}	$k_{L_4}\tau$ is the cycle of L_4	1
k_{L_5}	$k_{L_5}\tau$ is the cycle of L_5	1
k_{L_6}	$k_{L_6}\tau$ is the cycle of L_6	1
γ^{L_1}	L_1 to be scheduled γ^{L_1} times	1
γ^{L_2}	L_2 to be scheduled γ^{L_2} times	1
γ^{L_3}	L_3 to be scheduled γ^{L_3} times	1
γ^{L_4}	L_4 to be scheduled γ^{L_4} times	2
γ^{L_5}	L_5 to be scheduled γ^{L_5} times	2
γ^{L_6}	L_6 to be scheduled γ^{L_6} times	4
η^h	Heating efficiency	1
η^{ac}	AC efficiency	1
$\dot{M}$	Air flow rate	1 kg/s
$T^{h,min}$	Minimum heater-AC temperature	5°C
$T^{h,max}$	Maximum heater-AC temperature	32°C
R^{min}	Slew rate, temperature decrease	5°C
R^{max}	Slew rate, temperature increase	5°C
T^{init}	Initial room temperature	22°C
ε	ON/OFF status threshold	1°C

As we mentioned in the previous section, we assume in this chapter that the selling price signal and the purchasing price signal are different. Price forecasts for this simulation are given in Figures 44 and 45.

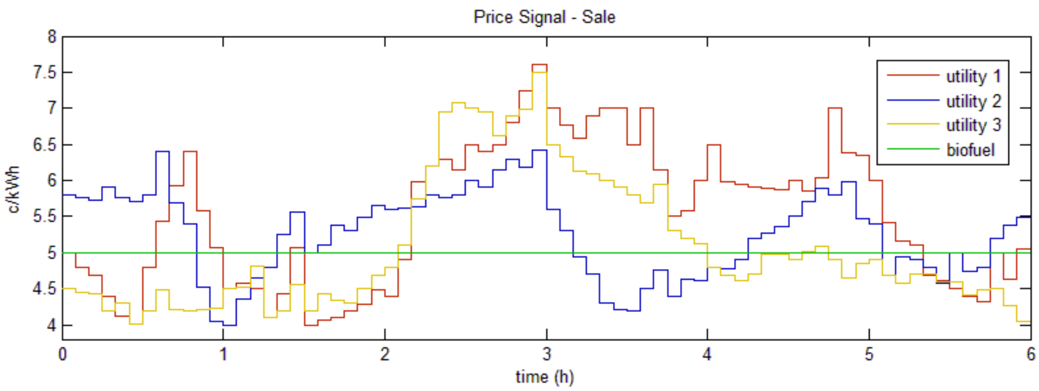


FIGURE 44: Price forecasts – sale – extension to several energy providers

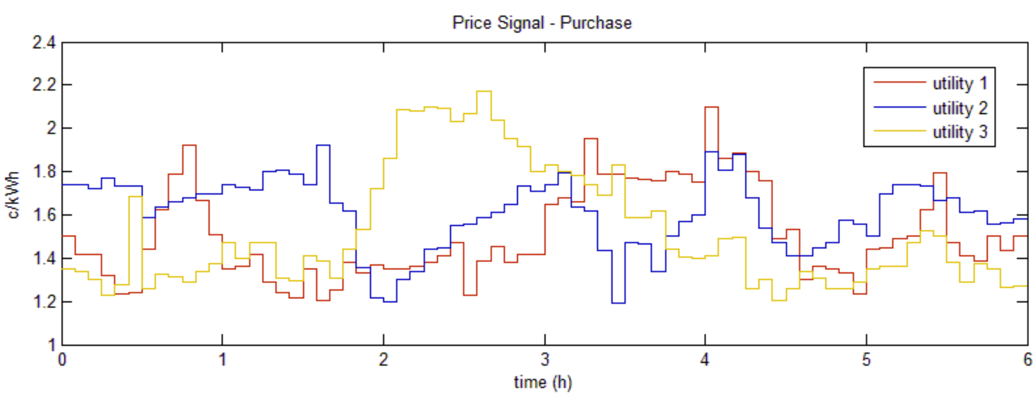


FIGURE 45: Price forecasts – purchase – extension to several energy providers

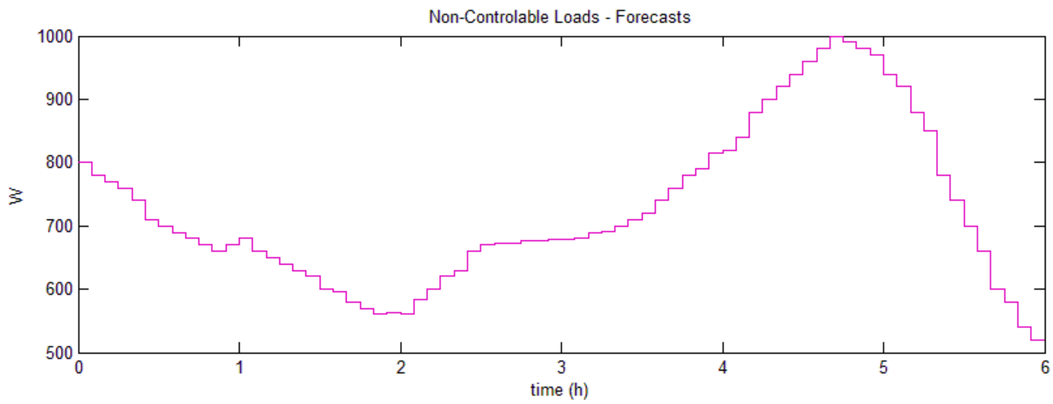


FIGURE 46: Non-controllable loads forecasts – extension to several energy providers

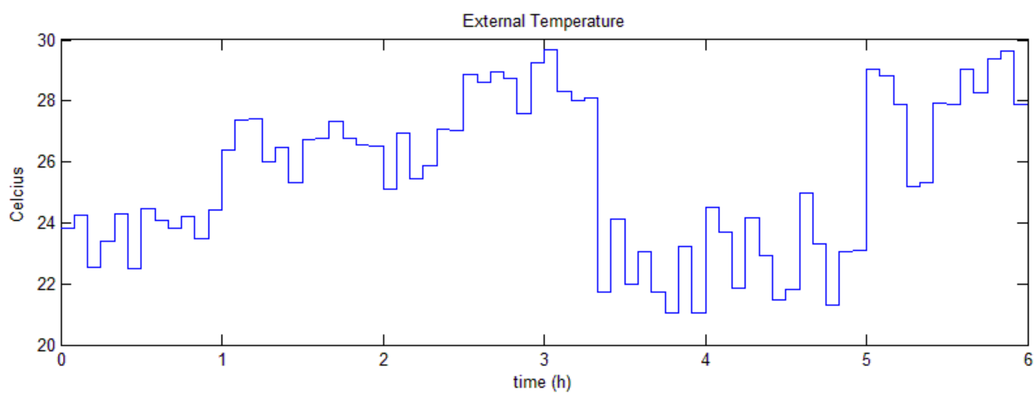


FIGURE 47: External temperature forecasts – extension to several energy providers

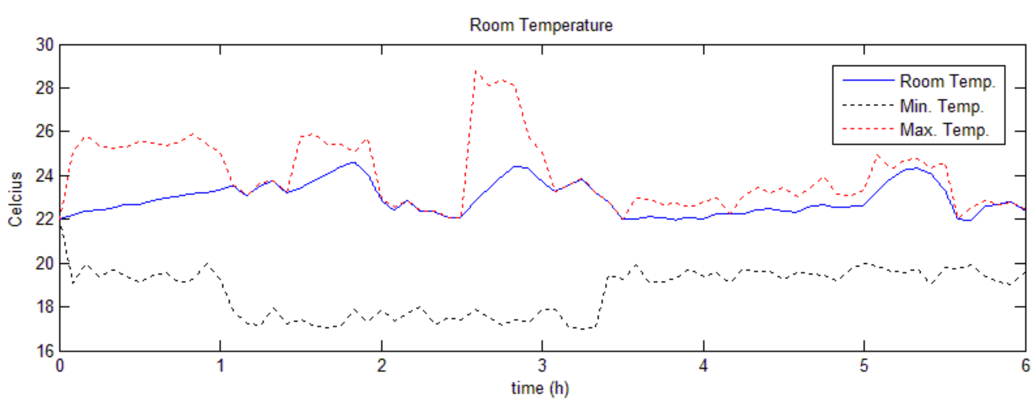


FIGURE 48: Room temperature – extension to several energy providers

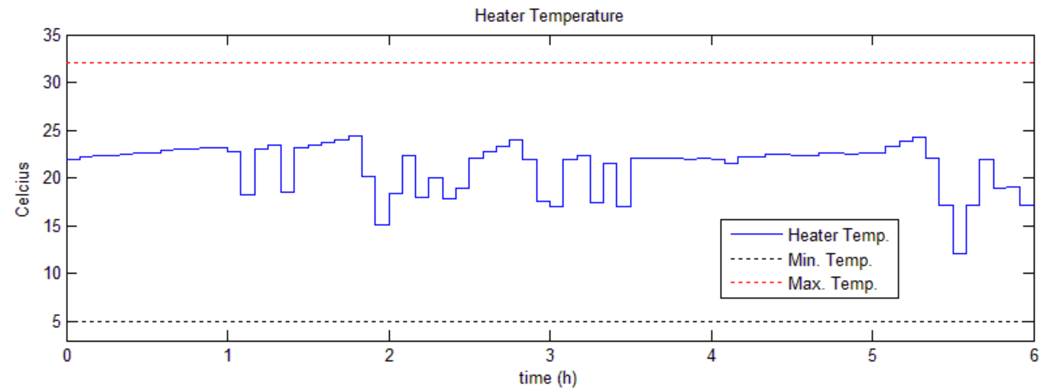


FIGURE 49: Heater temperature – extension to several energy providers

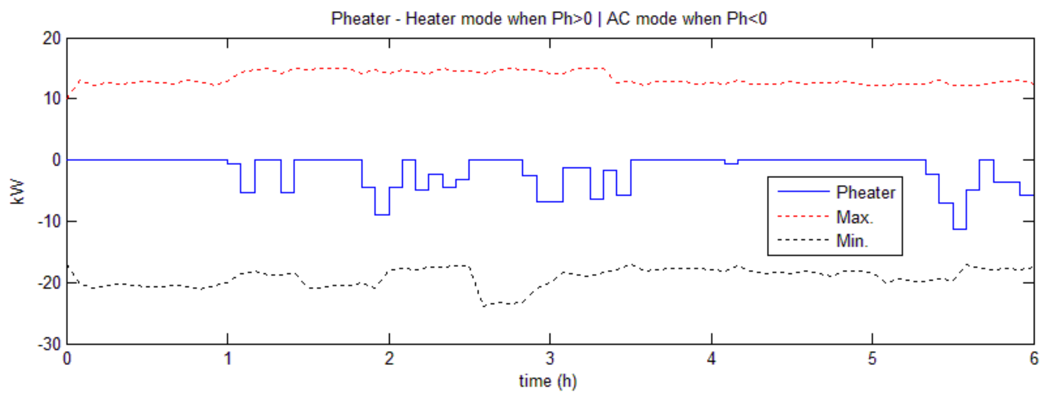


FIGURE 50: Heater power consumption – extension to several energy providers

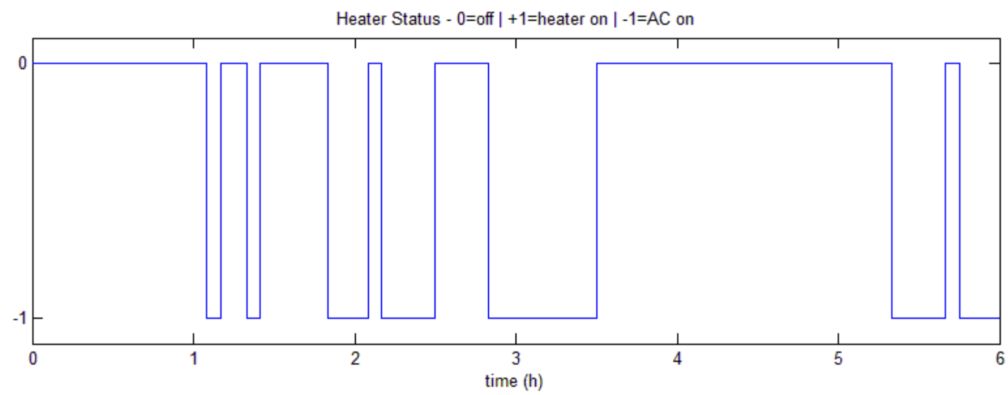


FIGURE 51: Heater status – extension to several energy providers

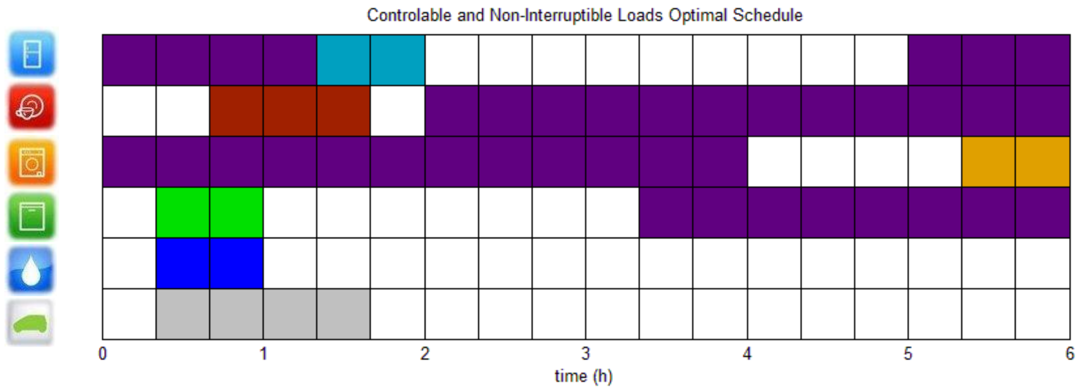


FIGURE 52: Optimal loads schedule – extension to several energy providers

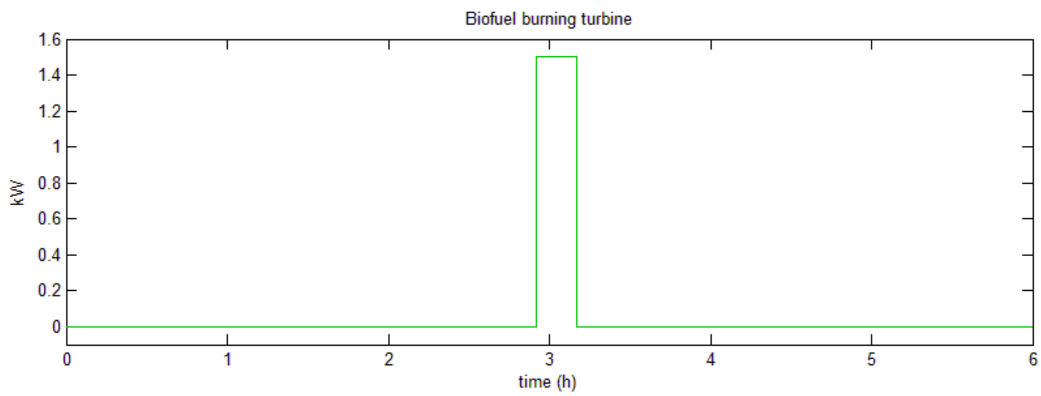


FIGURE 53: Biofuel-burning turbine schedule – extension to several energy providers

Notice in Figure 53 and 54 that the biofuel-burning turbine schedule and the storage schedule are not as predictable as they were in the previous simulations because we now have differentiated prices for selling and purchasing electricity.

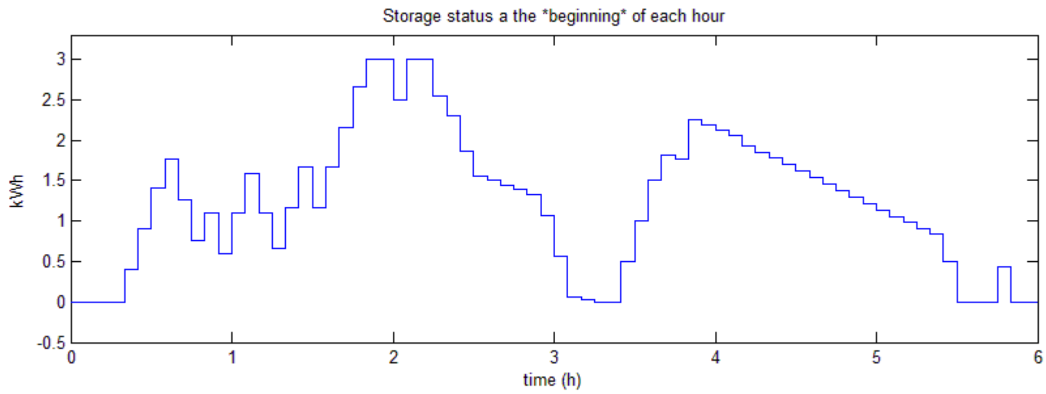


FIGURE 54: Storage status – extension to several energy providers

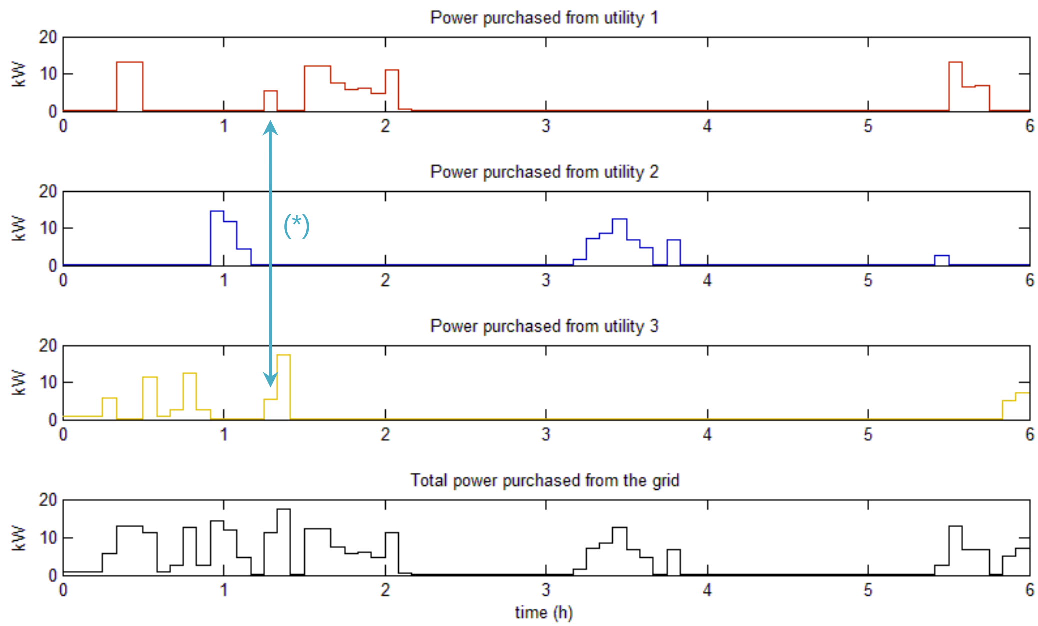


FIGURE 55: Power purchased from the grid – extension to several energy providers

Notice in Figure 55 that, logically, the energy provider chosen at a given time is always the cheapest one. The only exception is for an interval where providers 1 and 3 both sell electricity at exactly the same price (*).

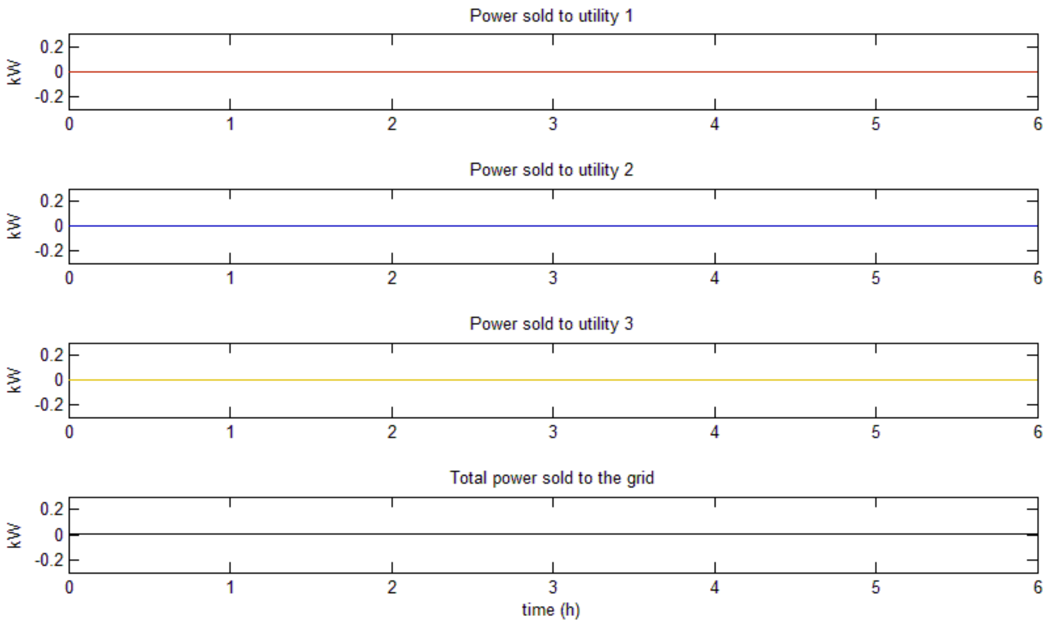


FIGURE 56: Power sold to the grid – extension to several energy providers

Figure 56 shows that in this scenario, electricity is never sold back to any energy providers because the purchasing prices are always lower than the selling prices. Then, there is no reason for the customer to buy and store electricity in order to sell it back later as he is certain to lose money in the process. It is also in the interest of the user to store his local production in excess, which is free, in order to use it later instead of selling it to the grid.

CHAPTER 8

APPLICATION OF THE GENERAL CASE TO REAL-TIME RESPONSE

8.1 - Introduction

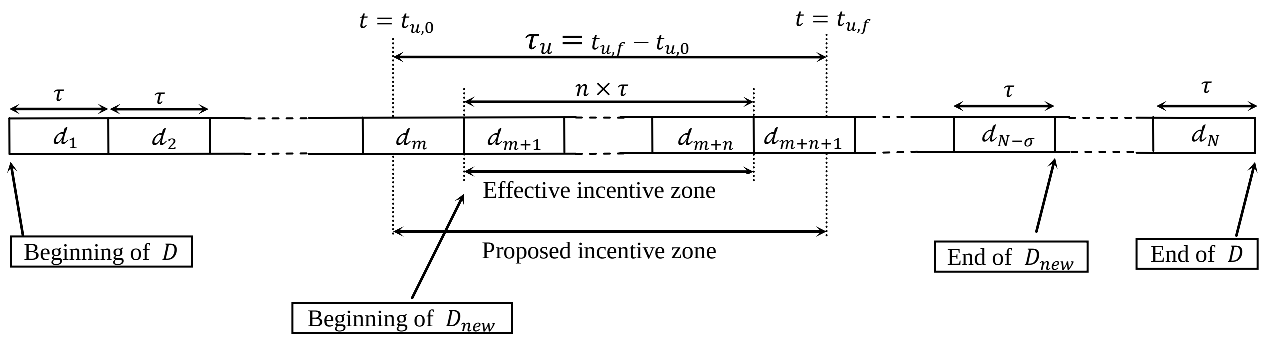
In this chapter, we consider a real-time scenario where the energy provider sends an incentive signal at $t = t_{u,0}$ to encourage residential customers to reduce their energy consumption for a given period of time. This situation can result of excessive demand or emergency conditions for example. The controller analyses the incentive signal and updates the optimum load schedule accordingly.

8.2 - Mathematical formulation as a linear programming problem

Assume that the optimization algorithm was run prior to the beginning of the period D. A subdivision $d = \{d_1, d_2, \dots, d_i, \dots, d_N\}$ was chosen, and there is an existing optimal schedule for the controllable loads.

The monitoring systems has turned on and off the non-interruptible loads accordingly since the beginning of D, and should keep doing so until its end. The heating/AC is run in real time by a separate control system which bases its decisions on both the optimal scenario forecast and the *actual* temperatures of the room and the outside. If the temperature forecasts and the thermal model of the house are reliable, the actual schedule of the heating/AC should be very close to the optimal schedule determined beforehand and based on the temperature forecasts.

At $t = t_{u,0}$, the provider sends a signal to the customer proposing an incentive to encourage the customer to reduce his consumption over a period $\tau_u = t_{u,f} - t_{u,0}$.



At time $t_{u,0}$, assume that we are somewhere in the time interval d_m. Then, after receiving this signal, we are going to re-evaluate our controllable load schedule from the beginning of d_{m+1} to the end of d_N. Call this interval D_{new}.

Any non-interruptible load which has been already turned on before receiving the provider signal has to complete its cycle and cannot be interrupted.

If non-interruptible loads were scheduled over D_{new}, this schedule is going to be re-examined to check if, with the new offer received from the provider, a better schedule can be found.

The signal received from the provider allows the customer to define a vector $[i_1\ i_2\ ...\ i_n]^T$, with i_j the incentive the provider is willing to pay per power unit reduced over d_j. Thus, if the customer reduces his power consumption by ΔP_j^g over d_j, the provider will pay him $i_j \times \Delta P_j^g$ for this time interval.

Knowing this, the user is going to solve again the mixed-integer linear programming problem (LP) over D_{new} conserving the same τ.

Note that if $m \not\equiv 0\ (mod\ q)$, meaning that m and 0 are not congruent modulo q, then σ intervals $d_{N-\sigma+1} \dots d_N$ are removed at the end of D_{new} so that $N - \sigma - m \equiv 0\ (mod\ q)$. If r is the remainder of the division algorithm of m by q, we have $\sigma = q - r$.

First, knowing the value of the parameter m, we need to detect which non-interruptible loads could potentially be rescheduled. Non-interruptible loads which were run before the beginning of d_{m+1} are now off picture; non-interruptible loads which started before the beginning of d_{m+1} cannot be interrupted; non-interruptible loads which were scheduled to start at or after the beginning of d_{m+1} can be rescheduled.

The initial optimization provided for each load L optimal values of the position parameters $\delta_1^L \dots \delta_{S_L}^L$. Given the value of m, we can determine for each load L what is the status of the position j with respect to d_{m+1}:

Case a: the period corresponding to the position j is to start at or after the beginning of d_{m+1}. This case occurs when $m + 1 \leq (j-1)q + 1$

Case b: the period corresponding to the position j started before the beginning of d_{m+1}, and ends after the beginning of d_{m+1}.

This case occurs when $(j-1)q + 2 \leq m + 1 \leq (j-1)q + 1 + (k_L - 1)q + (q-1)$

Case c: the period corresponding to the position j ended before the beginning of d_{m+1}.

This case occurs when $(j-1)q+1+(k_L-1)q+(q-1)+1 \leq m+1$

This allows us, for each load L, to scan all the possible position statuses, and to come up with the positions corresponding to the time slots starting at or after the beginning of d_{m+1}.

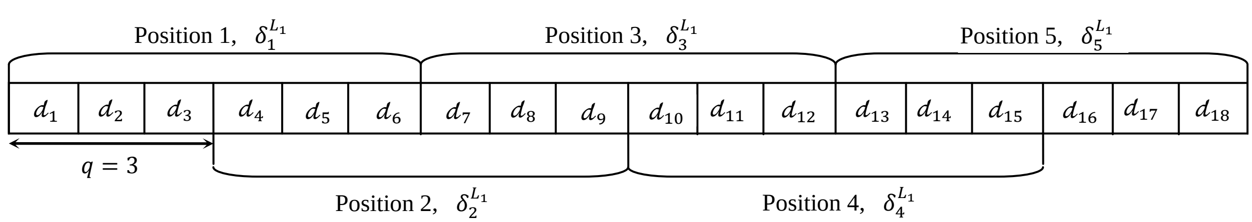
Once these positions are identified, the corresponding values of the position parameters δ_j^L are scanned in order to determine how many times the load L was scheduled in D_{new}. This allows us to determine how many times this load is to be rescheduled. This number is called γ_{new}^L.

Finally, the number of new possible positions between d_{m+1} and $d_{N-\sigma}$ for the load L is given by:

$$S_{L,new} = (N-\sigma-m) - k_L + 1$$

Example: for $N' = 6$, $q = 3$, $\ell = 1$, $k_{L_1} = 2$, $\gamma^{L_1} = 1$

From this we can derivate $N = 18$, $S_{L_1} = N' - k_{L_1} + 1 = 5$



Assume that before the beginning of d_1 the optimization problem was solved, and that the software determined that the best position to run L_1 was position 5.

Then $\delta_1^{L_1} = \delta_2^{L_1} = \delta_3^{L_1} = \delta_4^{L_1} = 0, \ \delta_5^{L_1} = 1$.

Assume that at some point over d_8 we receive an incentive signal from the provider. Then we have $m = 8$.

Let us determine the status of each position j:

$\underline{j = 1}$:
$(j-1)q + 1 + (k_L - 1)q + (q-1) + 1 = 7 \ \leq \ m + 1 = 9 \ \rightarrow$ case c

$\underline{j = 2}$:
$(j-1)q + 2 = 5 \ \leq \ m + 1 = 9 \ \leq \ (j-1)q + 1 + (k_L - 1)q + (q-1) = 9 \ \rightarrow$ case b

$\underline{j = 3}$:
$(j-1)q + 2 = 8 \leq \ m + 1 = 9 \ \leq \ (j-1)q + 1 + (k_L - 1)q + (q-1) = 12 \ \rightarrow$ case b

$\underline{j = 4}$: $m + 1 = 9 \ \leq (j-1)q + 1 = 10 \rightarrow$ case a

$\underline{j = 5}$: $m + 1 = 9 \ \leq (j-1)q + 1 = 13 \rightarrow$ case a

From the status analysis we see that only positions 4 and 5 are to be considered for rescheduling. Given that $\delta_4^{L_1} = 0, \ \delta_5^{L_1} = 1$, we know that the load L_1 is to be rescheduled only once over D_{new}: $\gamma_{new}^{L_1} = 1$.

Also, $m \not\equiv 0 \ (mod \ q)$, and $8 = 2 \times 3 + 2$, then $\sigma = q - r = 3 - 2 = 1$.

Finally, we are to reschedule L_1 once between d_9 and d_{17}.

Let us now come back to the general case, and call $\boldsymbol{X}$ the previous optimal solution to (LP). With the previous notations, we had:

$$\mathcal{C} = \widetilde{\boldsymbol{\mathcal{P}}}^T.\boldsymbol{X} = p_1^u P_{1,1}^{(g)} + \cdots + p_N^u P_{N',q}^{(g)} + P_{nom}^{bt} p^f \times (\delta_1^{bt} + \cdots + \delta_N^{bt})$$

Re-indexing the $P_{i,j}^{(g)}$ we have:

$$\mathcal{C} = \widetilde{\boldsymbol{\mathcal{P}}}^T.\boldsymbol{X} = p_1^u P_1^{(g)} + \cdots + p_N^u P_N^{(g)} + P_{nom}^{bt} p^f \times (\delta_1^{bt} + \cdots + \delta_N^{bt})$$

Let us call $\mathcal{C}'$ the cost corresponding to the power exchanged with the utility grid over D_{new} according to the pre-existing loads schedule:

$$\mathcal{C}' = p_{m+1}^u P_{m+1}^{(g)} + \cdots + p_{N-\sigma}^u P_{N-\sigma}^{(g)} + P_{nom}^{bt} p^f \times (\delta_{m+1}^{bt} + \cdots + \delta_{N-\sigma}^{bt})$$

Define $\widetilde{\boldsymbol{\mathcal{P}}}'$ and $\boldsymbol{X}'$ such that $\mathcal{C}' = \widetilde{\boldsymbol{\mathcal{P}}}'^T.\boldsymbol{X}'^T$**.**

$$\widetilde{\boldsymbol{\mathcal{P}}}' = \begin{bmatrix} p_{m+1}^u & \ldots & p_{N-\sigma}^u & P_{nom}^{bt} p^f \mathbf{1}_{\mathbb{R}^{N-\sigma-m}}{}^T & \mathbf{0}_{\mathbb{R}^{3(N-\sigma)+1+\sum_{i=1}^{\ell} s_{L_i,new}}}{}^T \end{bmatrix}^T$$

$$\boldsymbol{X}' = \begin{bmatrix} P_{m+1}^{(g)} & \ldots & P_{N-\sigma}^{(g)} & \delta_{m+1}^{bt} & \ldots & \delta_{N-\sigma}^{bt} & \mathbf{0}_{\mathbb{R}^{3(N-\sigma)+1+\sum_{i=1}^{\ell} s_{L_i,new}}}{}^T \end{bmatrix}^T$$

Let us now call $P_{m+1,new}^{(g)}, \ldots, P_{N-\sigma,new}^{(g)}$ the power exchanged over D_{new} after the re-evaluation of the customer's consumption following the reception of the provider signal.

The new cost $\mathcal{C}_{new}$ can be expressed as :

$$\begin{aligned}\mathcal{C}_{new} = \; & p_{m+1}^u P_{m+1,new}^{(g)} + \cdots + p_{N-\sigma}^u P_{N-\sigma,new}^{(g)} + P_{nom}^{bt} p^f \times \left(\delta_{m+1,new}^{bt} + \cdots + \delta_{N-\sigma,new}^{bt}\right) \\ & - \imath_1 \left(P_{m+1}^{(g)} - P_{m+1,new}^{(g)}\right) - \cdots - \imath_n \left(P_{m+n}^{(g)} - P_{m+n,new}^{(g)}\right)\end{aligned}$$

Then,

$$\mathcal{C}_{new} = (p^u_{m+1} + \imath_1)P^{(g)}_{m+1,new} + \dots + (p^u_{m+n} + \imath_n)P^{(g)}_{m+n,new} - \imath_1 P^{(g)}_{m+1} - \dots - \imath_n P^{(g)}_{m+n}$$
$$+ p^u_{m+n+1}P^{(g)}_{m+n+1,new} + \dots + p^u_{N-\sigma}P^{(g)}_{N-\sigma,new} + P^{bt}_{nom}p^f$$
$$\times \left(\delta^{bt}_{m+1,new} + \dots + \delta^{bt}_{N-\sigma,new}\right)$$

Let us define $\boldsymbol{\mathcal{I}} = [\imath_1\ \imath_2\ \dots\ \imath_n\ \mathbf{0}_{\mathbb{R}^{4(N-\sigma)+1+\sum_{i=1}^{\ell} S_{L_i,new}}}{}^T]^T$

Then we have :

$$\mathcal{C}_{new} = (\boldsymbol{\mathcal{I}} + \widetilde{\boldsymbol{\mathcal{P}}}')^T \boldsymbol{X_{new}} - \boldsymbol{\mathcal{I}}^T \boldsymbol{X}'$$

After receiving the provider signal, we then solve the following mixed-integer linear programming problem:

Minimize	$\mathcal{C}_{new} = (\boldsymbol{\mathcal{I}} + \widetilde{\boldsymbol{\mathcal{P}}}')^T \boldsymbol{X_{new}} - \boldsymbol{\mathcal{I}}^T \boldsymbol{X}'$	
Subject to	$\boldsymbol{A_{new}X_{new}} \le \boldsymbol{B_{new}}$ $\boldsymbol{A_{eq,new}X_{new}} = \boldsymbol{A_{eq,new}}$ $\forall i \in [\![m+1, N-\sigma]\!],\ \delta^{bt}_i \in [\![0,1]\!]$ $\forall i,j,\ \delta^{L_i}_j \in [\![0,1]\!]$ $\forall\ i \in [\![m+1, N-\sigma]\!],\ T^{r,min} \le T^{r,0}_i \le T^{r,max}$	(LP-new)

$\boldsymbol{X_{new}}, \boldsymbol{A_{new}}, \boldsymbol{B_{new}}, \boldsymbol{A_{eq,new}}, \boldsymbol{A_{eq,new}}$ are defined similarly to $\boldsymbol{X}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{A_{eq}}, \boldsymbol{B_{eq}}$ substituting $N - \sigma - m$ for N and $S_{L_i,new}$ for S_{L_i}.

(LP-new) gives us a new optimal schedule. If $\mathcal{C}_{new} < \mathcal{C}'$, this new schedule is adopted. At the same time, the system displays a message inviting the customer to reduce the consumption of his non-controllable loads until $t = t_{u,f}$.

8.3 - Simulation

8.3.1- Introduction

To conclude this chapter, we simulate a scenario involving both interruptible and non-interruptible loads in order to illustrate the mathematical theory discussed above. The system (LP-new) is solved for the home grid defined in Table 6.

Assume the following time constraints apply:

- Load 1 must run between 1:20 and 6
- Load 2 must run between 0 and 2
- Load 3 must run between 3:40 and 6
- Load 6 must run between 0 and 1:40

Assume solar module is off-line for this simulation.

TABLE 6
HOMEGRID PARAMETERS – CHAPTER 8

Symbol	Quantity	Assumed value
D	Time horizon	6h
N'	Number of time intervals – subdivision d'	18
τ'	Length of each time interval – subdivision d	20min
q	Subdivision factor	4
N	Number of time intervals	72
τ	Length of each time interval	5min
$\mathcal{u}$	Number of energy providers	1
p^f	Biofuel price	5.9 ¢/kWh
E^{init}	Energy stored at $t = 0$	0 kWh
E^{max}	Maximum capacity - storage device	3 kWh
R^c	Maximal charging rate - storage device	0.5 kWh
R^d	Maximal discharging rate - storage device	0.5 kWh
P^{bt}_{nom}	Nominal power - biofuel-burning turbine	1.5 kW
$\eta^{(w)}$	Wind turbine efficiency	0.20
$S^{(w)}$	Wind turbine eq. surface	$1m^2$
ℓ	Number of non-interruptible loads	6
P^L_1	Washing machine consumption	1 kW
P^L_2	Dryer consumption	1.5 kW
P^L_3	Dishwasher consumption	1 kW
P^L_4	Defrost cycle consumption	1.3 kW
P^L_5	Water heater consumption	2 kW
P^L_6	PHEV battery consumption	3 kW
k_{L_1}	$k_{L_1}\tau$ is the cycle of L_1	2
k_{L_2}	$k_{L_2}\tau$ is the cycle of L_2	3
k_{L_3}	$k_{L_3}\tau$ is the cycle of L_3	2
k_{L_4}	$k_{L_4}\tau$ is the cycle of L_4	1
k_{L_5}	$k_{L_5}\tau$ is the cycle of L_5	1
k_{L_6}	$k_{L_6}\tau$ is the cycle of L_6	1
γ^{L_1}	L_1 to be scheduled γ^{L_1} times	1
γ^{L_2}	L_2 to be scheduled γ^{L_2} times	1
γ^{L_3}	L_3 to be scheduled γ^{L_3} times	1
γ^{L_4}	L_4 to be scheduled γ^{L_4} times	2
γ^{L_5}	L_5 to be scheduled γ^{L_5} times	2
γ^{L_6}	L_6 to be scheduled γ^{L_6} times	4
η^h	Heating efficiency	1
η^{ac}	AC efficiency	1
$\dot{M}$	Air flow rate	1 kg/s
$T^{h,min}$	Minimum heater-AC temperature	5°C
$T^{h,max}$	Maximum heater-AC temperature	32°C
R^{min}	Slew rate, temperature decrease	5°C
R^{max}	Slew rate, temperature increase	5°C
T^{init}	Initial room temperature	22°C
ε	ON/OFF status threshold	1°C

8.3.2- Initial optimization

Similarly to what we did in chapter 6, let us determine the best schedule over the time period D.

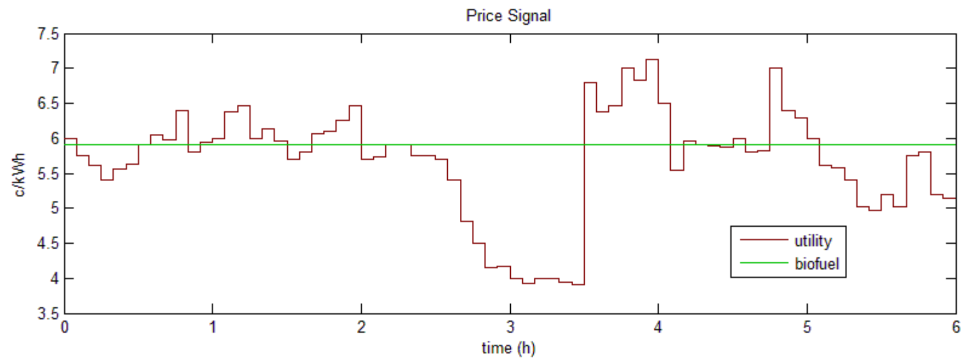


FIGURE 57: Price forecasts – Initial optimization

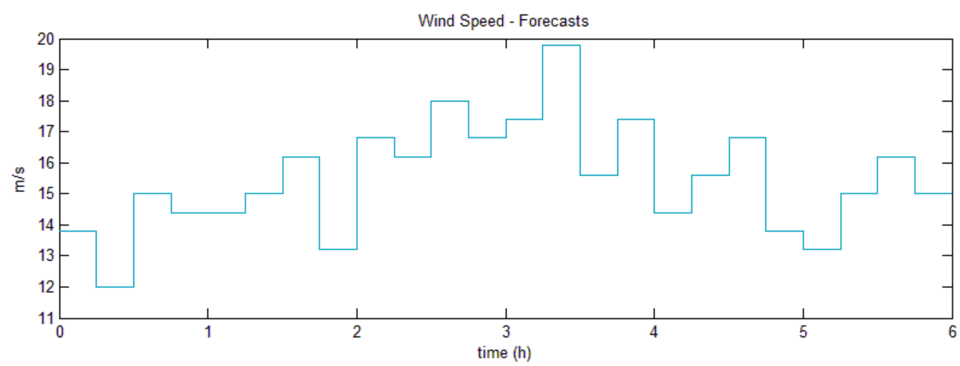


FIGURE 58: Wind speed forecasts – Initial optimization

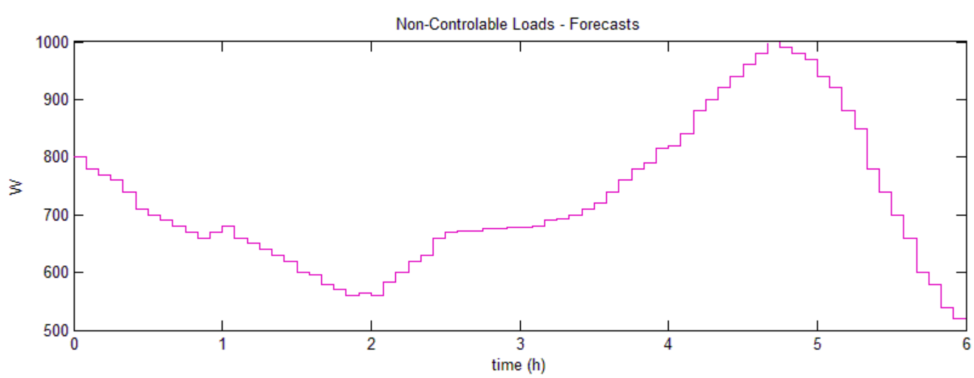


FIGURE 59: Non-controllable loads forecasts – Initial optimization

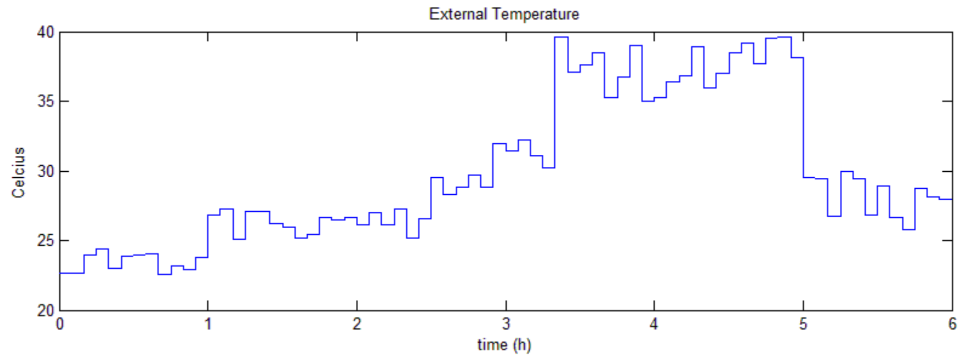


FIGURE 60: External temperature forecasts – Initial optimization

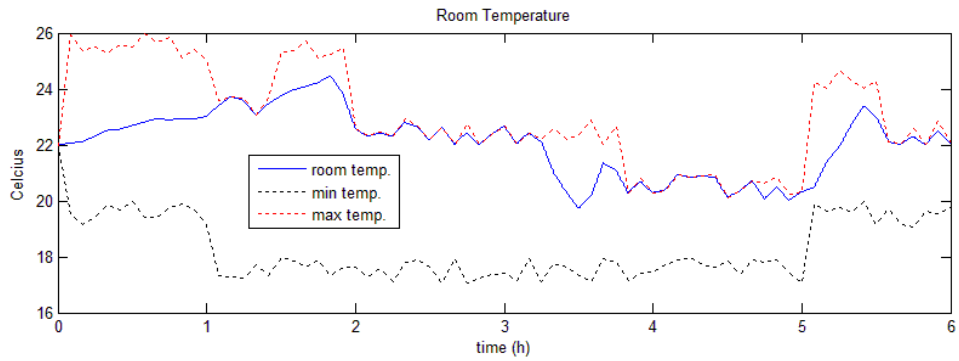


FIGURE 61: Room temperature – Initial optimization

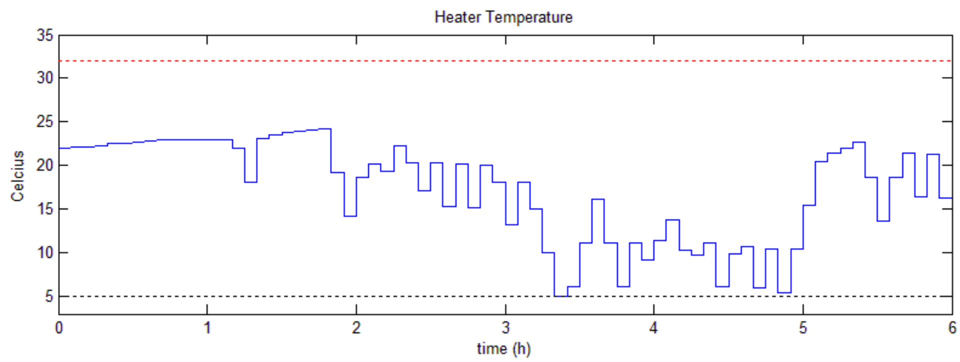


FIGURE 62: Heater temperature – Initial optimization

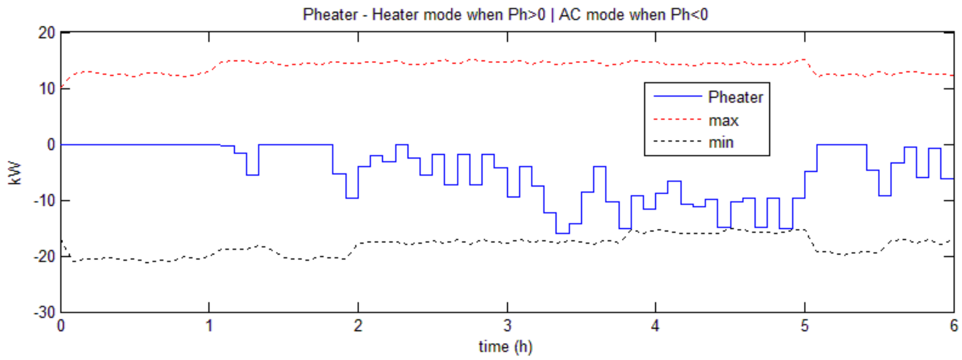


FIGURE 63: Heater power consumption – Initial optimization

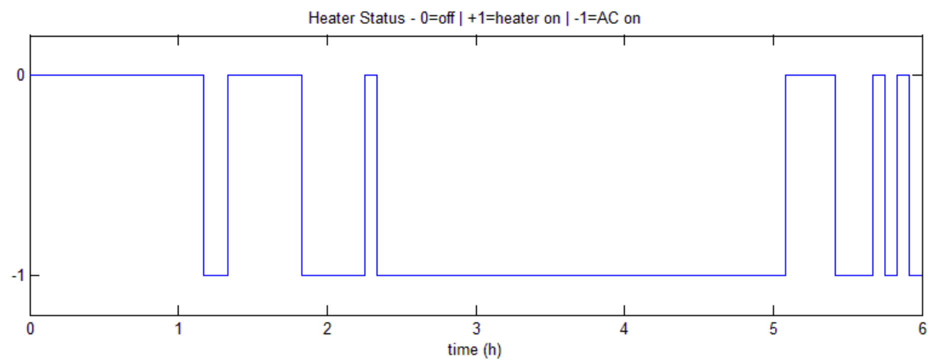


FIGURE 64: Heater status – Initial optimization

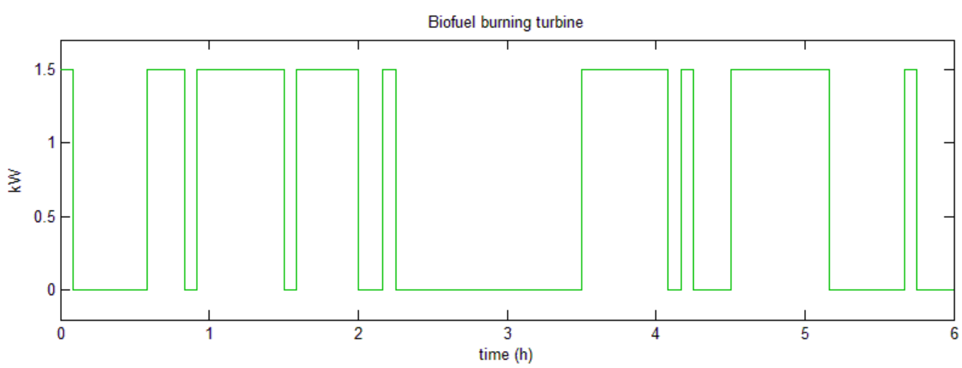


FIGURE 65: Biofuel-burning turbine schedule – Initial optimization

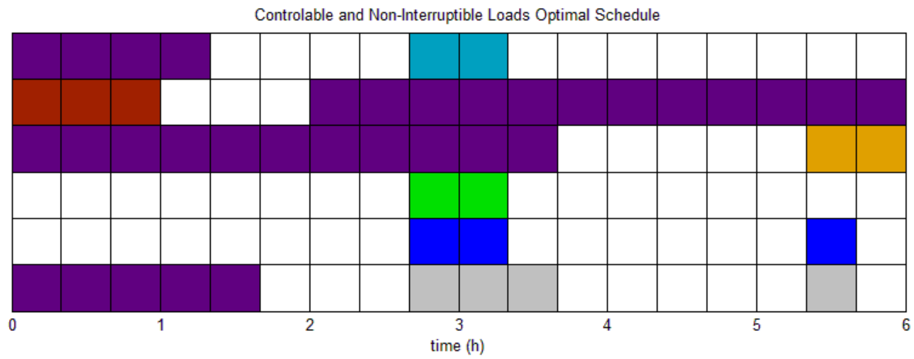


FIGURE 66: Optimal loads schedule – Initial optimization

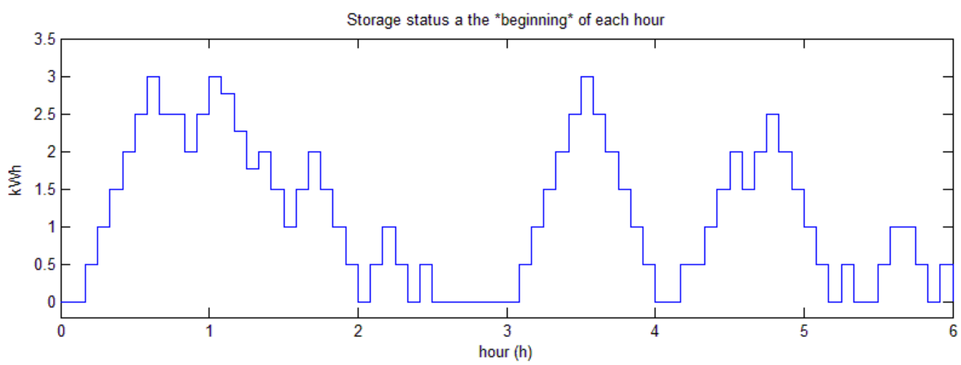


FIGURE 67: Storage schedule – Initial optimization

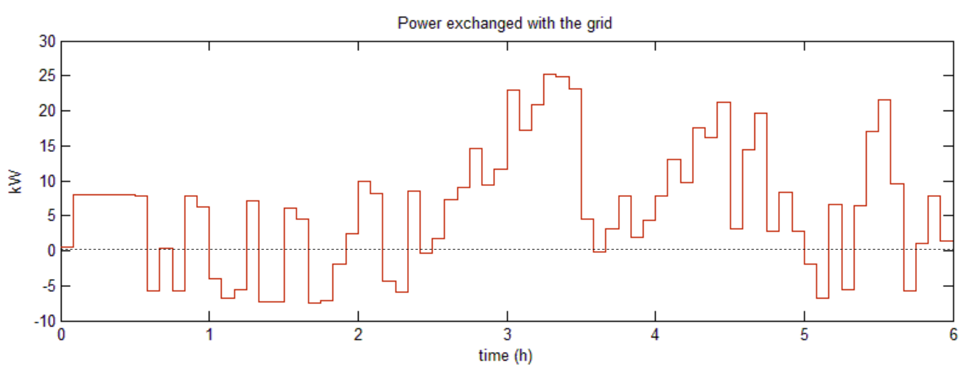


FIGURE 68: Power exchanged with the grid – Initial optimization

The price forecasted for this initial schedule is $ 8.31.

8.3.2- Real-time response

Assume now that at $t = 2{:}58$, the provider sends an incentive signal with $t_{u,f} = 4$.

Then $m = 36$, $\sigma = 0$.

Assume that $\forall\, j,\ i_j = 3\ \$/kW$ (constant incentive signal).

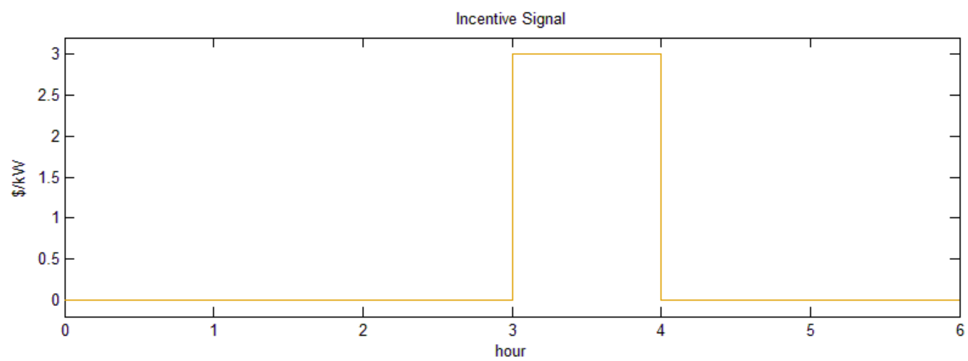


FIGURE 69: Incentive signal – Real time response

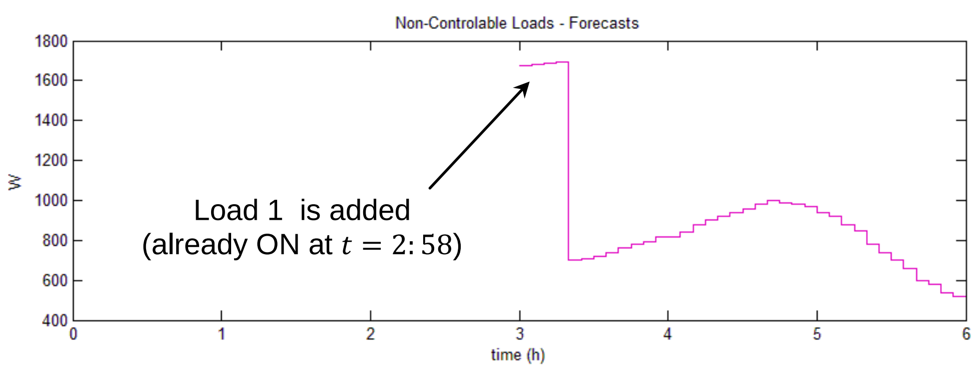


FIGURE 70: Non-controllable loads forecasts – Real time response

At $t = 2{:}58$, load 1 is already running and is expected to end at $t = 3{:}20$. Load 1 cannot be interrupted, then its remaining consumption is added to the non-controllable loads forecasts (Figure 70).

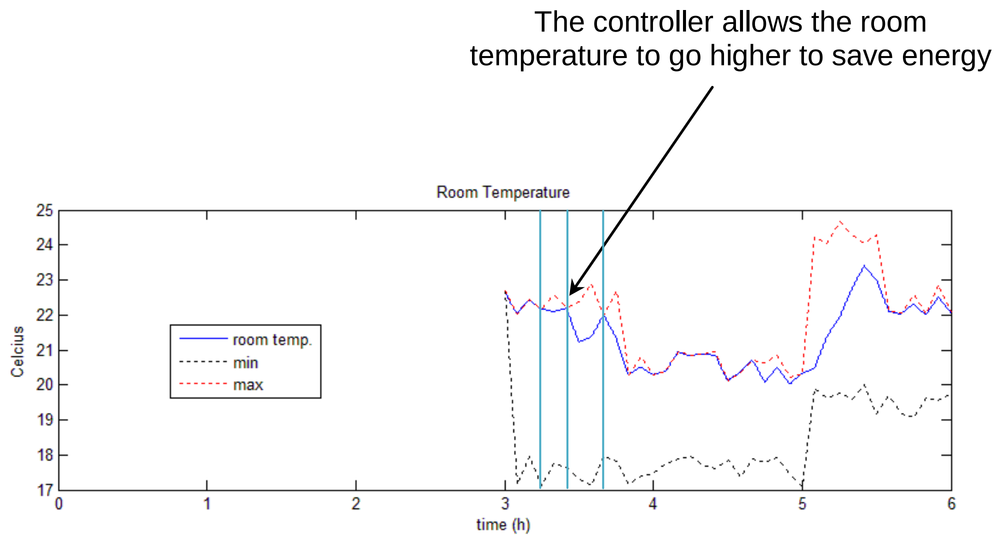


FIGURE 71a: Room temperature – Real time response

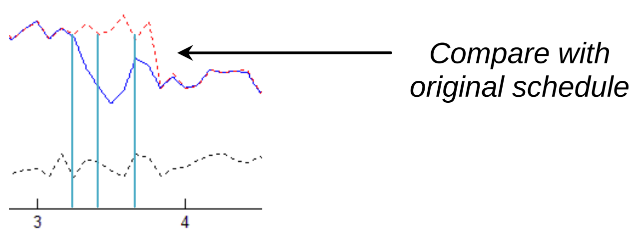


FIGURE 71b: Room temperature – Original schedule

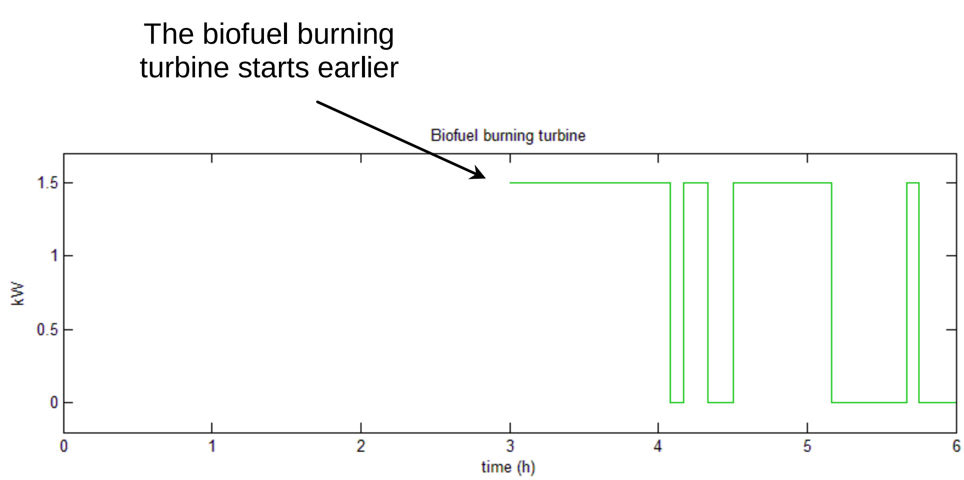


FIGURE 72: Biofuel-burning turbine schedule – Real time response

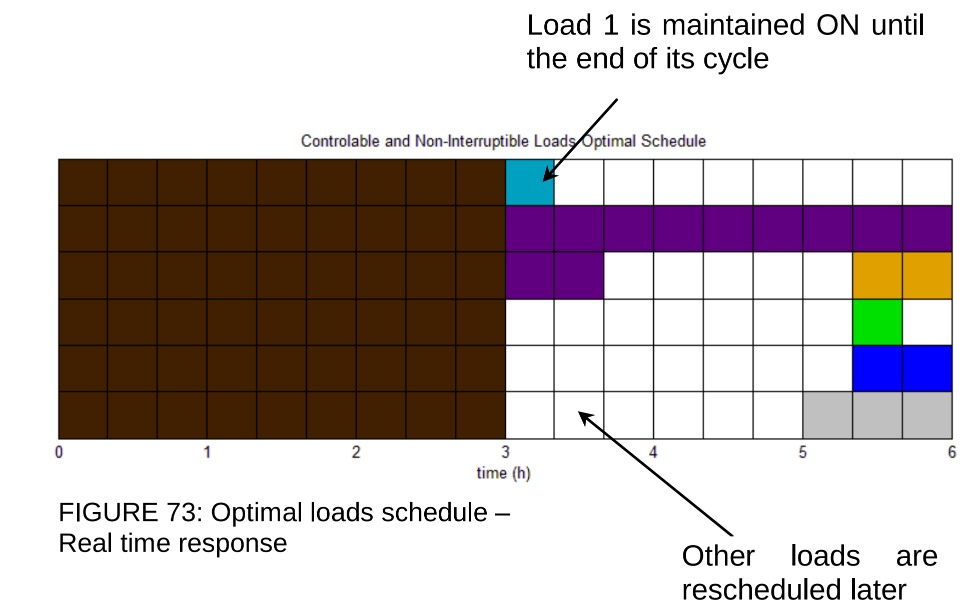


FIGURE 73: Optimal loads schedule – Real time response

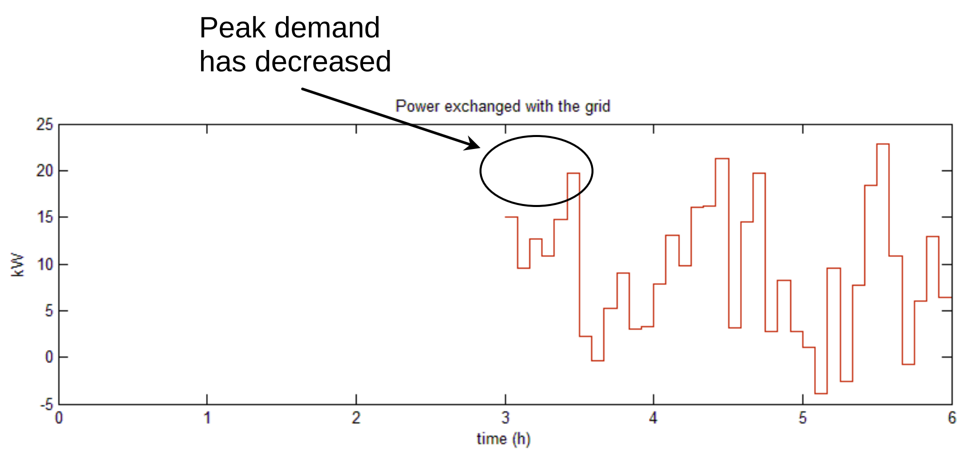


FIGURE 74: Power exchanged with the grid – Real time response

The original schedule corresponded to a cost of $ 8.31. The new schedule corresponds to a cost of $ 6.54. Then, the new scheduled is accepted.

CHAPTER 9

CONCLUSIONS

The objective of the present thesis was to formulate the optimization problem faced by a home energy manager controller, to implement the optimization controller, and to perform various simulations to evaluate the performance of the controller and the changes in energy use.

Several conclusions can be drawn based on this work:

Regarding system modeling, most of the relations between the unknown variables describing the system at a given time were shown to be linear. Equation (16) is the only equation we had to approximate so that the final system remains a linear function of the room temperature. But provided that τ is small enough, this approximation is fairly reasonable.

In this thesis we have assumed that the parameters of the model were constant. In future implementations, additional equations will be included to take into account the effects of the age and past history of the system, the number and frequency of charging and discharging cycles, and the external temperature, on parameters such as the efficiency $\eta^{(s)}$ of the solar panels, the maximum capacity E^{max} of the storage system, or the charging and discharging rates R^c and R^d.

Concerning the thermal model of the house, some state-estimation algorithm will be implemented in parallel to refine the empirical value of R_{eq} throughout time.

In future implementations, parameter values will still be maintained constant for a given optimization, but their values will be updated between each execution of the optimization algorithm. Note that the linearity, or non-linearity, of the potential effects of these external factors on the numerical values of the equation parameters will not affect the mathematical formulation of the optimization problem.

Based on the system modeled, we demonstrated that the optimization problem faced by a home energy manager controller could be formulated as a mixed-integer linear programming problem. This mathematical formulation encompasses both the physical constraints and the customer's comfort preferences. It is general enough to be used in both day ahead and real time response scenarios, and has been extended to the case of several energy providers.

Linear programming is a very important field of optimization as many practical problems can be expressed as linear programming problems in a wide range of fields (planning, production, transportation, etc.). This will allow us to use existing algorithms and numerical analysis methods applying to this class of problems in future implementations in order to improve the computational performance.

In this thesis, the controller was implemented in the Matlab environment and the algorithm used was based on the function *linprog*. Depending on the scenario, the algorithm was very fast (few seconds), or took up to several minutes to return a solution.

Scenarios limited to the heating/AC system presented in chapter 5 are the fastest to solve since they only involve continuous variables. In scenarios involving integer variables modeling non-interruptible loads, the computational time increases with the dimension of the solution space for the non-controllable loads schedule given by $\prod_{i=1}^{\ell}\binom{S_{L_i}}{\gamma^{L_i}}$. At some point, a multistart approach was necessary as the algorithm was sometimes unable to find a feasible point to begin with, or made an initial guess too far from the optimal point.

Future implementations will include pretreatment algorithms on the matrices $\boldsymbol{A}$, and $\boldsymbol{A_{eq}}$ to improve the condition numbers $\kappa(\boldsymbol{A})$ and $\kappa\left(\boldsymbol{A_{eq}}\right)$ in order to ensure a well-conditioned problem. Additional numerical simulations will be conducted to develop heuristic methods in order to formulate an educated guess for the initial solution point. This will increase the robustness and the resolution speed of the controller.

The proposed control architecture at the home level presents several advantages in terms of efficiency and privacy. This distributed control architecture avoids data transmission to controllers higher in the hierarchy, hence potential congestions on the communication network. With this architecture, the end-user keeps full control on all the devices connected to his home grid, and eventually decides what to turn on and off, and when. This architecture also protects the customer's ownership on his consumption data since all the calculations are handled locally.

This distributed intelligence could be embedded directly into the smart meter, with communication capacities in order to receive the price and weather forecast signals, and broadcast the optimal schedule to the devices connected to the home grid. However, because the meter is usually localized outside in order to facilitate access for the utility employees, this solution might present security and privacy weaknesses. Another solution would be to physically separate the metering function from the other functions. An electrical device would be maintained outside to take care of the metering function. At the same time, a local server would handle all the calculations. This server would be able to communicate with the home grid devices, including the energy meter, the loads and the various sensors, and could get the price and weather forecast signals from a dedicated web portal. The user could communicate with this server via a dedicated touch screen, or via other communication devices (phone, tablet computer, personal computer, etc.) in order to enter his comfort preferences or check his daily consumption.

The proposed pricing policy was shown to be very efficient in successfully achieving demand response. The use of day-ahead price signals allows the customer to optimally schedule his consumption in order to minimize his bill. Of course, the controller avoids scheduling loads during peak pricing period. In a dynamic pricing environment, energy providers are able to design price signals in such a way that they reflect the actual marginal prices. Our simulations showed that this eventually influences the consumption patterns of the customer. Future simulations will explore the implications of this result on demand peak shaving and loss reduction at the micro-grid and distribution level.

In chapter 8, we simulated a real-time scenario involving responding to an incentive signal. Our results show that the controller was successfully able to reschedule the controllable loads to respond to this signal, and increase the amount of power produced locally by the controllable generation unit. Other real-time scenarios will be considered in future implementations, involving updated weather forecasts signals, or signals resulting of emergency situations affecting the utility grid.

Notice that in both day ahead and real time, the energy consumption was optimized over a given temporal window based on price variations rather than price absolute values. This makes the proposed pricing policy fully compatible with government regulations imposing minimum and/or maximum price limits to energy providers.

Finally, the extension of the general case to a case with several energy providers showed that the controller was able to optimally determine, for a given pool of energy providers, when to buy electricity from each company based on its price signal.

As described in chapter 2, energy providers today have a strong market power on the retail market for residential customers. The implementation of this feature at the home level would allow deciding almost in real time from which energy provider to buy electricity from, and re-equilibrate the relation between the end-user and the energy provider.

In the simulations illustrating the multi-providers scenario, we assumed that the providers had made contracts at the provider level to exchange power between them, and that customers could not act as third parties in that process. However, our mathematical formulation is general enough to allow future simulations where the customer will not only be able to consume, produce and store energy, but also to buy power from provider i in order to sell this power immediately to provider k, or to another consumer, and make profits.

In order to minimize the customer's electricity bill, this thesis recognized the importance of the ability to shift the time when electric energy is transferred from the utility grid to the home grid. This was achieved through loads rescheduling, and using an energy storage system. Because the electricity time of use is directly related to the life habits of the customer and his comfort preferences, future work will develop this relation between the storage function and the ability to enforce the customers' consumption preferences. In particular, the economical value of storage at the residential level will be studied, and the consumer comfort preferences will be priced based on the costs of energy storage technologies.

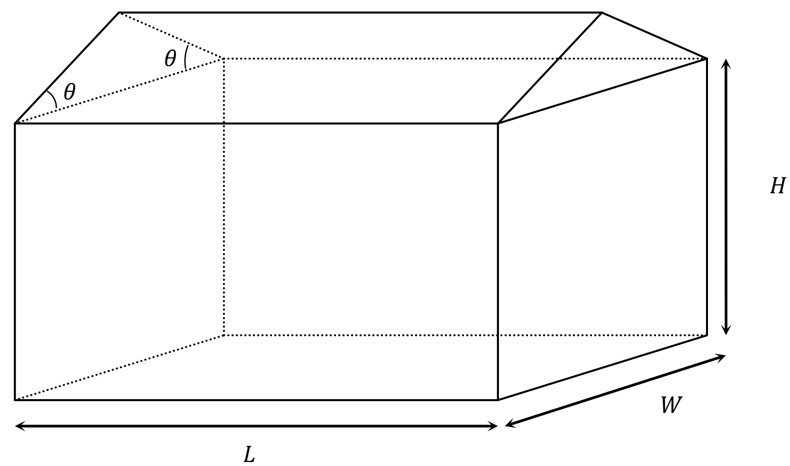
Concerning the distributed energy resources, the proposed mathematical formulation will allow future work to simulate the financial viability of a deployment of such technologies at the residential level. Our simulations show that distributed generation capabilities allow the customer to reduce his electricity bill. Additionally, the use of controllable units such as the biofuel-burning turbine featured in that thesis allows the user to respond to real-time request from the energy provider by increasing the local production.

In future work, the mathematical theory will be extended to take into account feed-in tariff policies, and simulations using estimations of price signal profiles for the coming years will allow conducting cost-benefit analysis and rate of return calculations of such technologies at the residential level.

Finally, the mathematical formulation based on the linear programming theory will be used to conduct future multi-agent based simulations at the micro-grid and utility grid levels. These simulations will explore the consequences of the proposed architecture on topics such as the resulting demand profile of a given network made of several independent agents, the distribution losses and their potential reduction, and the price elasticity of electrical energy.

APPENDIX A

GEOMETRY OF THE HOUSE



Assume for simulations:

$L = 30\ m$
$W = 10\ m$
$H = 4\ m$
$\theta = 40°$

The total wall area is $S_{wall} = 2HL + 2WH + \frac{WL}{cos(\theta)} + 2W^2 tan(\theta)$

Also assume the presence of 6 windows of a total area $S_{window} = 6\ m^2$

Let us call L_{wall} the thickness of the walls, L_{window} the thickness of the windows.

Assume for simulation $L_{wall} = 0.2\ m$ and $L_{window} = 0.01\ m$

APPENDIX B

CALCULATION OF THE EQUIVALENT THERMAL RESISTANCE

In the following, convective heat transfers between the walls and the air, and the windows and the air, are neglected.

Consider a section S of a material of thermal conductivity λ, with a thickness e.
The thermal resistance $R_{th}$of this material is defined by:

$$R_{th} \triangleq \frac{e}{\lambda S}$$

Define λ_{wall} the thermal conductivity of the walls (including the roof), and λ_{window} the thermal conductivity of the windows.
Assume for simulations $\lambda_{wall} = 0.038\, J/s/m/C$ and $\lambda_{window} = 0.78\, J/s/m/C$.

We have:

$$R_{wall} = \frac{L_{wall}}{\lambda_{wall} S_{wall}} \qquad R_{window} = \frac{L_{window}}{\lambda_{window} S_{window}}$$

Let call R_{eq} the equivalent thermal resistance of the house. By analogy with the equivalent resistance of two electrical resistances in parallel:

$$R_{th} = \frac{R_{wall} . R_{window}}{R_{wall} + R_{window}}$$

REFERENCES

[1] Electricité de France (EDF) - day/night TOU based demand response program, “Option heures pleines/heures creuses”, http://bleuciel.edf.com/abonnement-et-contrat/les-prix/les-prix-de-l-electricite/tarif-bleu-47798.html (Accessed June 25, 2010).

[2] Electricité de France (EDF) - Tempo program, “Option tempo”, http://bleuciel.edf.com/abonnement-et-contrat/les-prix/les-prix-de-l-electricite/tarif-bleu-47798.html (Accessed June 25, 2010).

[3] U.S. Department of Energy, “Smart Grid System Report” p.10, table 2.2, section1, July 2009, http://www.oe.energy.gov/SGSRMain_090707_lowres.pdf (Accessed June 25, 2010).

[4] HU, P.S., REUSCHER,T.,R, Federal Highway Administration, U.S. Department of Transportation, “Summary of travel trends: 2001 national household travel survey,” p. 30, December 2004, http://nhts.ornl.gov/2001/pub/STT.pdf (Accessed June 25, 2010)